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Welfare and efficiency in incomplete market economies with a single firm[☆,☆☆]

Egbert Dierker[a,b,*], Hildegard Dierker[a,b]

[a] Institut für Höhere Studien, Stumpergasse 56, A-1060 Wien, Austria
[b] Institut für Wirtschaftswissenschaften, Universität Wien, Hohenstaufengasse 9, A-1010 Wien, Austria

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ABSTRACT

In the quasilinear case, surplus maximization leads to constrained efficient Drèze equilibria. We investigate the question of whether surplus maximization can be useful beyond the quasilinear case.

We use two different surplus concepts, the equivalent and the compensating surplus. The first one is a utilitarian social welfare function and the second one a measure of inefficiency. We show that social welfare maximization can be at odds with constrained efficiency. In particular, a unique Drèze equilibrium can maximize welfare although it is not minimally constrained efficient. The Drèze equilibrium can also minimize welfare although it entails no efficiency losses.

We argue that the two surplus concepts should be used together and that they can help to distinguish between different Drèze equilibria on welfare and efficiency grounds.

1. Introduction

Drèze (1974) extends General Equilibrium Theory to incomplete markets with production. If markets are incomplete different shareholders of a firm rank production decisions differently. Drèze proposes a way to find a compromise among them. A systematic treatment of incomplete markets with production is given in the comprehensive book by Magill and Quinzii (1996).

Already Drèze (1974) presents examples which show that the coordination problem among several firms can lead to undesirable equilibria. Furthermore, if several goods per state are traded on spot markets inefficiencies can arise easily. To

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* Corresponding author at: Institut für Wirtschaftswissenschaften, Universität Wien, Hohenstaufengasse 9, A-1010 Wien, Austria.
E-mail address: egbert.dierker@univie.ac.at (E. Dierker).

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avoid these difficulties, we assume that there are a single firm and a single good per state. We consider two time periods, $t=0$ and $t=1$, and several states s at $t=1$. The good in the initial state 0 is called good 0.

Our framework corresponds to a partnership economy in the sense of §31 of Magill and Quinzii (1996). In a partnership economy, technologies exhibit constant returns to scale and original shares play no role.

We assume that the shares of the firm are the only tradable asset in the economy. The asset enables consumers to transfer wealth from period 0 to period 1. Consider the production plan $y=(y_0, \mathbf{y_1})$ where $\mathbf{y_1} \geq 0$ is the vector of outputs in the different states at $t=1$ and $y_0 \leq 0$ is the input needed at $t=0$. If consumer i purchases the share $\vartheta^i \geq 0$ then i pays $\vartheta^i \,|\, y_0 \,|$ units of good 0 in exchange for $\vartheta^i \mathbf{y_1}$. A *stock market equilibrium* obtains if the scale of production is such that the total demand $\sum_i \vartheta^i$ equals the supply which is normalized to 1.

A standard way to define Drèze equilibria relies on transfers of good 0 in order to enable the winners of a change of the production plan to compensate the losers. A *Drèze equilibrium* is a stock market equilibrium in which the production plan y satisfies the following criterion. The group $\mathcal{F}$ of final shareholders cannot change y infinitesimally and make infinitesimal transfers in terms of good 0 among its members such that every $i \in \mathcal{F}$ makes a first order utility gain. Infinitesimal share adjustments need not be taken into account because of the envelope theorem.

Hart (1976) strengthened this definition with the aim to take all Pareto improvements into account which can be achieved by changing a single firm's production plan. Hart introduced the concept of an *extended Drèze equilibrium* based on the idea that transfers of good 0 and a reallocation of shares of all firms can be made if a single firm's production plan is changed.

The assumption that transfers can be carried out in order to improve upon a stock market equilibrium places the emphasis on the consent of each individual shareholder. We call this the transfer based approach. Its goal is to describe a way which allows individuals to arrive at a unanimous decision.

The purpose of this paper is to explore efficiency and welfare properties of Drèze equilibria. We use the following terminology. The term efficiency refers to Pareto improvements in the absence of interpersonal utility comparisons. The word welfare refers to judgements that are made by an ethical outside observer on the basis of interpersonal utility comparisons.[1] More precisely, we shall assume that the observer makes cardinal utility comparisons which result in a utilitarian welfare function.

Instead of focussing on unanimous group decisions after transfers one can pursue other goals. Drèze (1974) aims at constrained efficiency. Consider a planner who cannot split assets to alleviate the market incompleteness but who can choose production plans, allocate shareholdings, and distribute the total endowment of good 0. An allocation is *constrained efficient* if this planner cannot make every consumer better off. Drèze equilibria can be characterized by the first order condition for constrained efficiency. If a stock market equilibrium is constrained efficient it must be a Drèze equilibrium.

In the quasilinear case, utility is transferable in the form of good 0 and the sum of the utility functions of a group can be used to measure its welfare. Furthermore, the maximization of society's total welfare entails constrained efficiency in this case. If one leaves the quasilinear setting, *utilitarian welfare maximization* presents a generalization that has been derived axiomatically. The reader is referred to the closely related papers by Dechamps and Gevers (1978) and Maskin (1978) and to the literature cited there.

Consider a set X of alternatives and an ethical observer who wants to order the elements of X by aggregating the individual preferences of a group N. In our setting, X represents the set of all stock market equilibrium allocations. Let (x, i) denote agent i in state x. The statement $U(x, i) \geq U(y, i')$ says that, from the perspective of the observer, individual i is at least as well off in equilibrium x as i' is in equilibrium y, where the function $U(\cdot, i)$ is required to represent i's preferences.

The observer ranks the pairs $(x, i) \in X \times N$ and obeys a set of general principles. Under the conditions of Maskin (1978), the observer's preference ordering is characterized as follows: alternative x is socially at least as desirable as alternative y if and only if $\sum_{i \in N} U(x, i) \geq \sum_{i \in N} U(y, i)$. As Maskin observes, the proof is essentially an application of Debreu's theorem on cardinal utility representations. The function $\sum_i U(\cdot, i)$ describes the observer's preference over the set X of alternatives. Due to its cardinal nature, the utilitarian rule lends itself most easily to a setting with von Neumann–Morgenstern utilities.

Consider an interior stock market equilibrium in an economy with von Neumann–Morgenstern utilities and let consumer i receive an infinitesimal increase of good 0 at his equilibrium consumption $\hat{x}^i$. Suppose that the observer is indifferent with respect to i's identity so that i's infinitesimal utility gain raises social welfare by an amount that is independent of i. Then the von Neumann–Morgenstern utility representations U^i can be chosen such that $\partial_0 U^i(\hat{x}^i) = 1$ for every i. On can show that the stock market equilibrium is a Drèze equilibrium if it is a critical point of the utilitarian welfare function $\sum_i U^i$. Hence, a Drèze equilibrium is a candidate for a welfare maximum.

In Section 2.3, we describe an example of an economy with von Neumann–Morgenstern utilites and a unique Drèze equilibrium which is not constrained efficient although it maximizes the welfare function $\sum_i U^i$. In the example, no stock market equilibrium is constrained efficient. Since constrained efficiency is out of reach one is led to weaken this efficiency requirement.

To reduce the power of the planner who makes transfers we rule out that transfers can be invested so that future consumption depends only on the production plan. This leads to the concept of minimal efficiency as defined in Dierker et al. (2005).

[1] The so-called first and the second welfare theorem do not fall into this category.