Journal of Economic Dynamics & Control 33 (2009) 1–14

Contents lists available at ScienceDirect

Journal of Economic Dynamics & Control

journal homepage: www.elsevier.com/locate/jedc

Wealth distribution and aggregate time-preference: Markov-perfect equilibria in a Ramsey economy

Paul Pichler, Gerhard Sorger *

Department of Economics, University of Vienna, Hohenstaufengasse 9, A-1010 Vienna, Austria

ARTICLE INFO

Article history:
Received 27 April 2007
Accepted 20 April 2008
Available online 14 May 2008

JEL classification:
C63
C73
D31
O41

Keywords:
Ramsey–Cass–Koopmans model
Strategic saving
Markov-perfect Nash equilibrium
Aggregate time preference
Projection method

ABSTRACT

We study Markov-perfect Nash equilibria (MPNE) of a Ramsey–Cass–Koopmans economy in which households are aware of their influence on prices. The Ramsey conjecture fails to hold such that households other than the most patient one own positive wealth in the steady state. This confirms results that have been derived in the same model using an open-loop equilibrium concept. In contrast to the competitive and the open-loop equilibrium, the steady state of the MPNE depends on the utility functions of the households. Since the MPNE cannot be determined analytically, a high-order least squares projection method is employed.

1. Introduction

We consider the standard infinite-horizon growth model under the non-standard assumption that households realize their influence on the factor prices. Using a high-order least squares projection method, we solve for Markov-perfect Nash equilibria (MPNE) of that model. The analysis focuses on the wealth distribution and the aggregate time-preference rate in steady states generated by MPNE, but there is also some discussion of the transition dynamics towards the steady state.

A well-known shortcoming of the standard Ramsey–Cass–Koopmans model with price-taking households (see Ramsey, 1928; Cass, 1965; Koopmans, 1965) is its prediction about the long-run distribution of wealth. If all households share the same time-preference rate, then it follows that this distribution depends on the initial endowments. If, on the other hand, households differ with respect to their time preferences, the long-run wealth distribution is independent of initial conditions but it is degenerate in the sense that only the most patient household owns a positive amount of capital; see Becker (1980). In Sorger (2002) it was argued that this feature of the model contains a seed for its own destruction. If the steady state is asymptotically stable (as it is under reasonable assumptions), then it follows that, over time, the capital market becomes a monopoly. This, in turn, renders one of the basic assumptions of the model questionable, namely that of price-taking households. To circumvent this problem, Sorger (2002) suggested to assume that the households realize their market power (at least when the economy is close to the steady state). Under this assumption it was shown that the long-run distribution of wealth is not necessarily degenerate and that impatient households may be willing to maintain positive

* Corresponding author. Tel.: +43 1 4277 37443; fax: +43 1 4277 9374.
E-mail address: gerhard.sorger@univie.ac.at (G. Sorger).

0165-1889/$ - see front matter © 2008 Elsevier B.V. All rights reserved.
doi:10.1016/j.jedc.2008.04.006

capital holdings forever. These results about the steady states of the *strategic* Ramsey–Cass–Koopmans model were later generalized by Becker (2003) and Sorger (2008). Moreover, Becker and Foias (2007) derived sufficient conditions for the asymptotic stability of the steady state in that model.

The studies by Sorger (2002, 2008), Becker (2003), and Becker and Foias (2007) are all based on the concept of an open-loop Nash equilibrium of the strategic Ramsey–Cass–Koopmans model. This equilibrium concept is as close as possible to the usual competitive model while still containing that crucial strategic element that turns out to be responsible for the emergence of a non-degenerate long-run wealth distribution. On the other hand, the open-loop Nash equilibrium concept assumes that all households can credibly commit to all their future consumption and labor supply decisions. This means in particular that the households' decisions are not state contingent. As the motivation for the suggestion by Sorger (2002) is only compelling if market forces have already steered the economy close to a degenerate wealth distribution, an equilibrium concept based on state-contingent policy functions seems to be more appropriate than one that is based on full commitment. The most obvious choice for such an equilibrium concept is MPNE. The property of Markov-perfection assumes only a limited form of commitment to future consumption and labor decisions and captures an ongoing and, hence, much stronger form of strategic interaction between the players than the open-loop equilibrium. The first contribution of the present paper is therefore to check whether and how the main insights from the above mentioned studies carry over to the concept of MPNE.

We find that in an MPNE there exists a unique steady state, that this steady state is asymptotically stable, and that the long-run wealth distribution is typically non-degenerate. As far as the steady state wealth distribution is concerned, it follows therefore that the main insights from the literature cited above do carry over to the MPNE in a qualitative sense. Surprisingly, we also find that the quantitative differences between the MPNE steady state and the open-loop steady state are not as big as one might expect. One aspect, however, in which the MPNE differs significantly from the open-loop equilibrium (and from the competitive steady state as well) is that the steady state generated by the MPNE depends on the utility functions of the households. More specifically, we find that a reduction of a household's intertemporal elasticity of substitution leads, ceteris paribus, to an increase of that household's steady state capital stock and to a reduction of the other households' steady state capital holdings. In a competitive equilibrium or an open-loop equilibrium, on the other hand, the location of the steady states is independent of all preference parameters except for the time-preference factors. The intuitive explanation for the influence of the utility functions on the steady state is that, in a MPNE, each household takes into account the reaction of its opponents to changes in the state of the economy. Since this reaction depends obviously on the form of the utility function, it follows that preference parameters like the elasticity of intertemporal substitution affect the location of the steady state. The present paper also emphasizes the strategic substitutability of the capital holdings of different households. If one household reduces its capital stock, the interest rate increases and the wage rate decreases. This creates a substitution effect which induces the other households to increase their own capital stocks. Of course, there is also an income effect associated with the change in factor prices and we show that the relative strength of income and substitution effects differs between rich and poor households.

A second contribution of the present paper is the study of the aggregate time-preference rate in the strategic Ramsey economy. We measure the aggregate time-preference rate by the real interest rate generated in an MPNE. Gollier and Zeckhauser (2005) have shown that, in every Pareto- optimal allocation, the aggregate time-preference rate is a weighted average of the individual time-preference rates with the weights being proportional to the individual tolerances for consumption fluctuations. This result does not automatically carry over to our framework, because households have market power and the equilibrium allocation is therefore not Pareto optimal. For a realistic calibration of the model, however, we can show that the aggregate time-preference rate differs only slightly, though systematically from the prediction made by Gollier and Zeckhauser (2005). The systematic bias results from two sources. First, the strategic interaction of the households exerts upward pressure on the interest rate for the same reason that prices in an oligopoly exceed competitive prices. Second, proportionality of the weights to the individual tolerances for consumption fluctuations is perturbed by the potentially different degrees of commitment power that the households have. A household with a low elasticity of intertemporal substitution does not react quickly to changes in factor prices such that it can be said to have higher commitment power. Conversely, a household with a high elasticity of intertemporal substitution can easily be influenced by its opponents, because it reacts quickly to any changes in factor prices. Using this interpretation of the elasticity of intertemporal substitution, we argue that the aggregate time-preference rate can be expressed as a weighted average of the individual time-preference rates with more weight given to the households that have higher commitment power.

A further contribution of our paper is about the dynamics generated by MPNE. We show that the transition dynamics are typically non-monotonic, and that both the speed and the direction of adjustment are highly affected by the households' elasticities of intertemporal substitution. Finally, the paper contributes to the literature on computational techniques, since it presents an efficient algorithm for the numerical computation of the MPNE in the strategic Ramsey–Cass–Koopmans model. Since the model cannot be solved for a steady state without determining the equilibrium strategies, standard perturbation methods (like local linearization) cannot be used. We therefore solve the model using projection methods as suggested by Judd (1992). More precisely, we implement a least squares projection method with a high degree of approximation. It turns out that, unlike in many growth models, the least squares approach performs very well in our setup. It displays excellent convergence properties and delivers highly accurate solutions.

The assumption that households possess market power is of course quite uncommon in the literature. We would like to emphasize that we do *not* propose to replace the competitive model by one with strategically interacting households as