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Effects of consumption strategy on wealth distribution on scale-free networks

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ABSTRACT

We investigate wealth distribution on scale-free networks with different consumption strategies. We indicate that nonlinear consumption function can lead to exponential wealth distribution with a power law tail in accordance with empirical data. In addition, we suggest that anti-degree preference and consumption promotion can make distribution more equal. This provides an effective and practical way to optimize the equality of wealth distribution.

1. Introduction

As simplified description for complex systems, complex networks have attracted much interest in the last decade [1–5]. In these research works, power law distribution is observed in several phenomena, ranging over Internet, transportation systems, social networks, etc [6]. Especially, in the economic field, power law is obtained in the income distribution, world trade networks, firm sizes, stock markets, etc [7–11]. So the scale-free (SF) network, which has a power law degree distribution $P_{\text{deg}}(k) \sim k^{-\gamma}$, is a reasonable description for economic and social systems.

One of the most important properties of an economic system is wealth distribution. Generally, we use cumulative wealth distribution $P_{>}(w) \equiv \int_{w}^{\infty} p(w')\mathrm{d}w'$ to depict wealth distribution. In capitalist economies, it shows a very peculiar functional form and shares some universal features, even if quantitative differences exist across various economies. In the range of high income, Vilfredo Pareto [12] finds that cumulative wealth distribution is power law, i.e. $P_{>}(w) = \int_{w}^{\infty} p(w')dw' \sim w^{-\gamma}$, whereas, in the low and middle income region, Robert Gibrat [13] first notices that it is well characterized by a log-normal distribution. However, Kalecki [14], Yakovenko and Rosser [15] points out that the log-normal distribution has a time-dependent variance, thus it is not stationary. Later on, by analogy with the Boltzmann–Gibbs distribution of energy in physics, it is shown that the probability distribution of wealth is exponential $P(w) = ce^{w/T}$ for certain classes of models with interacting economic agents [15,16]. Also, exponential distribution is a stationary solution of the Fokker–Planck equation [15]. Thus, the exponential distribution is more suitable for describing the lower class than the log-normal one. In practice, it may be difficult to distinguish between these two distributions. It is shown that the log-normal distribution can approximate the exponential one in a certain range of parameters [17]. However, the exponential function has fewer fitting parameters than the log-normal one, so it is preferable for fitting of the data. The log-normal/exponential distributions with a power law tail are frequently observed in real world economic phenomena. This empirical behavior has been confirmed by the economic data of Australia [18], the US [19], Japan [20], etc.

In order to explain this distribution, several microscopic models of wealth dynamics have been proposed [15,21,22]. In particular, we focus on the model proposed by Bouchaud and Mézard (BM model) [23], which takes both the stochastic

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property and interaction between agents into consideration. The wealth of N agents is governed by the following stochastic differential equation:

$$\dot{w}_i(t) = \eta_i(t)w_i(t) + \sum_{j(\neq i)} J_{ij}w_j(t) - \sum_{j(\neq i)} J_{ji}w_i(t), \tag{1}$$

where $w_i(t)$ is the wealth of agent i at time t, the $\eta_i(t)$'s are independent Gaussian variables of mean m and variance $2\sigma^2$ (due to random speculative trading such as market investments), and J_{ij} is the element of an interaction matrix describing the fraction of agent j's wealth flowing into agent i (account for transactions between i and j). Bouchaud and Mézard investigate a mean-field limit of this equation and show wealth distribution in large wealth range is power law type [23].

Later on, some researchers have investigated BM model on complex networks [8,21,24,25]. This provides a new way to study BM model. Several previous papers have focused mainly on the influence of topology of the interaction networks [24,25]. Most of them only consider the simplest wealth dynamics: the consumption of an agent is proportional to its wealth and flows equally to its neighbors. In this case, wealth distribution that similar to real data on scale-free networks is not reproduced. Although networks with various kinds of topology have been explored, different types of consumption strategy still remain undiscussed. In fact, as we will show, the consumption strategy, such as consumption function and preference, can have dramatic effects on wealth distribution.

In this paper, we focus on the effects of consumption function and preference on wealth distribution. We investigate the cases of linear and nonlinear consumption function theoretically. In linear case, we find that wealth distribution can be estimated by the out degree of transfer matrix. An exponential distribution with a power law tail is not obtained with linear consumption function, whereas it is reproduced in the nonlinear case. So we indicate that the nonlinear consumption function maybe the possible origin of this type of distribution in real economy. In addition, we discover that for both cases, anti-degree preference can make the distribution more even. In particular, for nonlinear case, enhancing the consumption of rich people is efficient to obtain a more equitable distribution. We discuss the time to achieve stationary state by simulations. It is shown that, promoting consumption can speed up the process to get steady. We perform numerical simulations on Barabási–Albert (BA) networks [2] to verify these results.

2. Model description

We use a finite undirected network $G = (V, E)$ as the underlying structure of our model. Here $V = \{1, 2, \ldots, N\}$ is the set of nodes and $E = \{(i, j)|i, j \in V\}$ is the set of edges. The adjacent matrix of G is $A = (a_{ij})_{N\times N}$, where a_{ij} takes the value of 1 if $(i, j) \in E$ and 0 otherwise. We denote the degree of node i as k_i, which can be given by $k_i = \sum_j a_{ij}$. Denote $w_i(t)$ as the wealth of agent i at time t.

In our model, we still use the framework of BM model. We will discuss different types of consumption function and preference. In economy, consumption function reflects the relation between the consumption and wealth of an agent. Consumption preference describes how an agent's consumption flows to its neighbors.

By analyzing this model, we mainly investigate the effects of different types of consumption function and preference, which is not discussed in previous papers on BM model. This new research point can provide practical and effective methods to adjust real economy.

2.1. Linear consumption function

First we discuss the condition of linear consumption function. We assume that the consumption of an agent is proportional to its wealth. Thus the consumption function has the form:

$$c(w_i(t)) = Cw_i(t) \tag{2}$$

where $c(w_i(t))$ is expenditure of agent i at time t and C is a constant in [0, 1].

In real economics, the expenditure of an agent does not flow equally to its neighbors. Here we use $P_{ij} = \{P_{ij}\}_{N\times N}$ to describe the fraction of agent j's expenditure flowing into agent i. We use the following form of P_{ij} to characterize the weighted consumption preference of agent j:

$$P_{ij} = \frac{a_{ij}k_i^{\alpha}}{\sum a_{pj}k_p^{\alpha}}. \tag{3}$$

Here the summation is taken over all the neighbors of agent j. In this form, if the parameter α is positive, expenditure tends to flow into its neighbors who have more economic transactions, and otherwise if α is negative. The tunable parameter α just reflects the consumption preference of each agent. So in Eq. (1), $J_{ij} = CP_{ij}$.

Discretize equation (1), we obtain

$$w_i(t+1) = (\eta_i(t) + \sigma^2)w_i(t) + \sum_{j(\neq i)} J_{ij}w_j(t) - \sum_{j(\neq i)} J_{ji}w_i(t). \tag{4}$$

Notice that Eq. (1) is described in the Stratonovich sense, which generates the second term proportional to σ^2 when discretized.