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Effects of introduction of new resources and fragmentation of existing resources on limiting wealth distribution in asset exchange models

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ABSTRACT

Pareto law, which states that wealth distribution in societies has a power-law tail, has been the subject of intensive investigations in the statistical physics community. Several models have been employed to explain this behavior. However, most of the agent based models assume the conservation of number of agents and wealth. Both these assumptions are unrealistic. In this paper, we study the limiting wealth distribution when one or both of these assumptions are not valid. Given the universality of the law, we have tried to study the wealth distribution from the asset exchange models point of view. We consider models in which (a) new agents enter the market at a constant rate (b) richer agents fragment with higher probability introducing newer agents in the system (c) both fragmentation and entry of new agents is taking place. While models (a) and (c) do not conserve total wealth or number of agents, model (b) conserves total wealth. All these models lead to a power-law tail in the wealth distribution pointing to the possibility that more generalized asset exchange models could help us to explain the emergence of a power-law tail in wealth distribution.

1. Introduction

A century ago, an Italian social economist Pareto collected and studied data of distribution of income across several European countries. He observed that 80% of the income is in 20% of the hands and the distribution of income has a power-law tail, *i.e.* $p(x) \propto x^{-1-\nu}$, where $p(x)$ is the probability that an individual has an income x. The exponent ν is called the Pareto index. The exponent measured by him for different kingdoms and countries varied between 1.1 to 1.7. The distribution of wealth also shows a similar behavior. The validity of the Pareto law has been questioned and reexamined many times. In modern times, the Japanese, Australian and Italian personal income distribution have been shown to demonstrate a log normal distribution for lower incomes coupled with a power-law tail [1–3]. For wealth distribution, the distribution of wealth in rich Indian families has a power-law tail [4]. The same feature is observed in the wealth of Hungarian aristocratic families [5]. Even for ancient Egyptian society, it has been conjectured that the wealth distribution had a power-law tail [6]. Empirical studies on the data of the distribution of income and wealth in modern USA and UK show a power-law tail as well [7]. All these studies suggest the existence of a power-law tail of wealth distribution and income distribution in different societies in different parts of the world and it seems to be true in older as well as modern times. However, the value of the exponent changes in different societies. We will attempt to explain wealth distribution from the viewpoint of asset exchange models in this paper.

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Given that the universality of the Pareto law is so robust, cutting across economies which follow different financial system, and even across time, there must be a simple explanation for this feature. Several models have been proposed from this viewpoint. There are attempts to use ideal-gas like models which recover these features [8]. Models in analogy with directed polymers in random media [9] have been proposed. Generalized Lotka–Volterra dynamics [10] and a stochastic evolution equation which incorporate trading as well as random changes in prices of investments [11] have also been proposed.

Trading is an economic activity which is common to all systems in all countries and has been so from time immemorial. Thus asset exchange models which are the simplest models of economic transaction should give us an explanation of the Pareto law. There have been several attempts in this direction [12–16]. We try to study it from the viewpoint of asset exchange models. In the literature, two types of asset exchange model have been studied extensively. In these models, there is neither consumption nor production of wealth. One of models is called the yard-sale model (YS) and other is known as the theft-fraud (TF) Model [17]. In the YS model, the amount at stake is a certain fraction of the wealth of a poorer agent while in the TF model, both agents put a certain fraction of their wealth at stake. However, none of these models reproduces the power-law distribution of wealth. The YS model leads to condensation of wealth in the hand of one agent asymptotically while the TF model which is an ideal-gas like model gives us an exponential distribution of wealth. In this context, several approaches have attempted to reproduce a power-law tail in wealth distribution starting from asset exchange models. Sinha showed that a modified YS rule in which the poorer player wins with higher probability leads to a power-law distribution of wealth [18]. In a previous paper, we showed that mixing the above two models leads to a power-law tail in resultant wealth distribution [15]. Several other variants such as introducing altruism in YS and TF models [19], introducing a saving propensity in TF type models [13,14] etc. have been studied.

A common and rather unrealistic feature of these models is that the rules do not allow total wealth in a society to fluctuate, nor the number of players in the society to change in time. It is clear that changes in working population makes an impact on the wealth distribution of the country. It is also clear that the true GDP (Gross Domestic Product) of the world has increased over time. Thus the total wealth is not conserved. In fact, a projection of real GDP growth is obtained by summing the estimates of the percentage changes in: increased labor inputs, increased capital inputs and productivity growth [20]. Thus it is clear that increases in labor supply will increase the real GDP (the proportionality constant is elasticity of output with respect to labor). In most societies, this number keeps growing. (Of course, there are exceptions to this rule of thumb. In older times, catastrophic events such as drought, earthquake, plague have wiped out populations. In modern economies, even a decrease in the labor supply is possible. Populations in countries such as Russia, Italy, Ukraine etc. are reducing and economists are discussing its consequences [21]. However, this is a new trend and its outcome will be apparent only after a decade or so [22].) Labor supply depends on demography. Kitov in has postulated that the GDP growth rate depends on relative change in the number of people with a specific age (9 years in the USA) [23]. Thus a linear relationship between the growth of real GDP and growth of the (working) population is a reasonable assumption. We are not taking into account the impact of a decreasing labor participation ratio on the economy in this paper which is a new phenomenon. We are not taking into account the impact of productivity or fitness of the agents or influx of new capital by agents already in the system. We will deal with the only the effect that the total wealth also keeps increasing since an increasing population discovers newer sources of income. We will systematically analyze the impact of an increase in the number of agents in the Yardsale model of asset exchange.

Wealth often splits at the higher end of the spectrum. Rich people have descendants who become independent agents in their own right. Similarly, large corporations split into entities which function independently. There is a fragmentation of wealth when people have children or companies split. Apart from the fact that large wealth is difficult to manage, there are social and legal pressures which encourage division of wealth of rich people. The society at large, resents a concentration of wealth in hands of some people leading to income inequalities. The government, on the other hand, wants to discourage monopolies from the perspective of encouraging economic efficiency and puts in measures such as antitrust act, ceiling act, quotas [24,25] etc. These measures affect rich people more than the poor.

There have been previous attempts to take into account these factors. Slanina has given a model with nonconserved wealth but a conserved number of agents. He models the wealth distribution in analogy with inelastically scattering particles and reports a power-law distribution with a Pareto index in the interval [1,2] depending on a free parameter introduced in the model [26]. There is an attempt to take into account the splitting of wealth between agents. R. Coelho et al. introduced the family-network model for wealth distribution in societies. Here, they assume fragmentation of wealth of older agents among their neighbors. This agent reappears with zero age and gets linked to two randomly selected agents that have wealth greater than a minimal value q. The wealth q is taken away from the wealth of these selected agents and it is redistributed in a random and preferential manner in society. This model leads to a Pareto-like power-law tail for the upper 5% of the society. The Pareto index in this model is found to be 1.8 [27]. But this model is static, and the total wealth and the number of agents are conserved. Lee and Kim introduced the model with a nonconserved number of agents similar to the model of a growing network. In that model, the number of agents increases linearly with the time but the model does not consider any exchange of assets, *i. e.* flow of money between the agents. The wealth production for any agent is due to intrinsic ability to produce wealth. This model leads to a power-law tail of the wealth distribution [28]. As we argued, given the universality of the law, we feel that one should be able to obtain it within the paradigm of asset exchange models. Despite its faults, we believe that a YS model is a good model of financial transactions. We make an attempt to explore the 'design space' of asset exchange models, in particular that of the YS model, by introducing changes in capital and labor. Taking the YS model as a