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Inequalities of wealth distribution in a conservative economy

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Abstract

We analyze a conservative market model for the competition among economic agents in a society with conserved total wealth. A minimum dynamics ensures that the poorest agent has a chance to improve its economic welfare. After a transient, the system self-organizes into a critical state where the wealth distribution have a minimum threshold, with almost no agent below this poverty line. Also, very few extremely rich agents are stable in time. Above the poverty line the distribution follows an exponential behavior. The local solution exhibits a low Gini index, while the mean field solution of the model generates a wealth distribution similar to welfare states like Sweden.
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1. Introduction

The study of the distribution of the income of workers, companies and countries started a little more than a century ago when Italian economist Vilfredo Pareto investigated data of personal income for different European countries and found a power-law distribution that seems not to depend on different economic conditions of the countries. In 1887, in his book *Cours d'Economie Politique* [1] he asserted that in all countries and times the distribution of income and wealth follows a power-law behavior where

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the cumulative probability $P(w)$ of people whose income is at least w is given by $P(w) \propto w^{-\alpha}$, where the exponent α is named Pareto index. This universal power law is now called Pareto law and it was shown by Pareto himself that the exponent α for several countries was $1.2 \leqslant \alpha \leqslant 1.9$. However, recent data suggest that Pareto's distribution does not agree with observed data over the middle range of income, while it is generally accepted that it provides a good fit to the distribution of high range of income. Many other distributions of income were proposed, supported by international empirical data, and the distribution behaves in a log-normal, exponential or Levy way [2] in the middle income range. For instance, there are data from Japan and USA that are fitted by a log-normal distribution in middle range plus a power law for high income one [3], data from United Kingdom income and wealth fitted by an exponential law (middle range) and a power law (high range) [4] and data from Brazil [5] worker's income and enterprise capital that are fitted by a log-normal plus a power law for high range of income. [1]

One of the goals of the study of wealth distribution is to determine the social inequalities implied in a given model of economic exchanges. Many ways to quantitatively measure the fairness of a given income distribution have been proposed in the economic science. Among them, the Gini index is well known and frequently used. In brief, for any given wealth density function $p(x)$, with mean ρ and a given number $v \in [0,1]$, $P(\leqslant v) = \int_0^v p(x)\,dx$ represents the cumulative fraction of population with wealth below v. The Lorenz function, $L(v)$, is defined as the fraction of the total wealth which is attributed to the poorest $100v$ percentage of the society. It is easy to show that $L(v) = \frac{1}{\rho} \int_0^v xp(x)\,dx$ and the Gini index can be then defined as $G = 2 \int_0^1 (v - L(v))\,dv$ [6]. It assumes values between 0 percent (perfect equality) and 100 percent (perfect inequality).

Recently, we developed a simple parameter free model for the competition among different economic agents within a conservative economy. Results for the wealth distribution has been published elsewhere [7] and also considering the effect of a small world network [8]. Within this model we obtained an exponential Gibbs-style distribution for the wealth, with a finite lower threshold or poverty line, that is characteristic of models with extremal dynamics. The idea of a society that take measures in order to improve the situation of the most impoverished is compatible with the propositions of John Rawls, in his book "A Theory of Justice" [9], directed towards an inventive way of securing equality of opportunity as one of the basic principles of justice. He asserts that *no redistribution of resources within . . . a state can occur unless it benefits the least well-off* and this should be the only way to prevent the stronger (or richer) from overpowering the weaker (or poorer).

In the following, we review the assumptions and principal results of the model and we present some new calculations concerning the time evolution of the system. The determination of the Gini index and the comparison with wealth distribution of several countries will be presented in Section 2 along with the conclusions.

The Conservative Exchange Market Model (CEMM) [7] is a simple macroeconomic model that consists of a one-dimensional lattice with N sites and periodic boundary

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[1] As 500 maiores sociedades anônimas do Brasil, Fundação Getulio Vargas, www.fgv.br/IBRE/CEAE/arq/x500M.xls