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Properties of wealth distribution in multi-agent systems of a complex network

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ABSTRACT

We present a simple model for examining the wealth distribution with agents playing evolutionary games (the Prisoners' Dilemma and the Snowdrift Game) on complex networks. Pareto's power law distribution of wealth (from 1897) is reproduced on a scale-free network, and the Gibbs or log-normal distribution for a low income population is reproduced on a random graph. The Pareto exponents of a scale-free network are in agreement with empirical observations. The Gini coefficient of an ER random graph shows a sudden increment with game parameters. We suggest that the social network of a high income group is scale-free, whereas it is more like a random graph for a low income group.

1. Introduction

Complex networks can describe a wide range of systems of high importance, ranging from nature to society and biological systems. Since the discovery of small-world behavior [1] and the scale-free property [2], complex networks have attracted continuous attention [3]. By representing the agents of a given population with vertices, and the contacts between agents with edges, network theory provides a natural framework for describing population structure [4]. For example, well-mixed populations can be represented by complete (fully connected, regular) networks and spatially structured populations can be associated with regular networks. Recently, much empirical evidence from real social networks has revealed that they are associated with a scale-free, power law degree distribution, $d(k) \sim k^{-\gamma}$ with $\gamma_{\text{actor}} = 2.3 \pm 0.1$ for a movie actor collaboration network [5], $\gamma_{\text{science}} = 2.1$ and 2.5 for a science collaboration graph [6], $\gamma_f = 3.5 \pm 0.2$ and $\gamma_m = 3.3 \pm 0.2$ for females and males in human sexual contacts [7], etc.

It is well known that even in developed countries, it is common for 40% of the total wealth to be owned by only 10% of the population. The distribution of wealth is often described using 'Pareto' tails (1897), which decay as a power law of the large wealth [8]. The rest of the people, with low income, follow a different distribution, either Gibbs or log-normal, [10,11]:

$$P(W) \sim \begin{cases} W^{-\upsilon}, & \text{for } (W \geq W_c), \\ \exp(-\lambda W), & \text{for } (W < W_c). \end{cases} \quad (1)$$

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where $P(W)$ is the probability of finding an agent with wealth greater than W. The exponent υ is called the Pareto exponent, and λ denotes a scaling factor. The value of the Pareto exponent was found to vary between 1 and 3 for both individual wealth and company sizes [12–17]. Studies on real data show that the high income group does indeed follow the Pareto law, with $\upsilon \approx 1.6$ for the USA [12], $\upsilon = 1.8 \sim 2.2$ for Japan [13,14], $\upsilon = 2.0 \sim 2.3$ for the UK [11], $\upsilon \approx 1.0$ for Japanese firms [18], and $\upsilon \approx 0.9$ for India [19]. The value of υ changes with the countries, the scale of the individual or company, and the statistical methods applied in the investigations.

The Gini coefficient was developed by the Italian statistician Corrado Gini in 1912 to measure the inequity of income distribution. It is defined as a ratio with values between 0 and 1: the numerator is the area between the Lorenz curve of the distribution and the uniform distribution line; the denominator is the area under the uniform distribution line [20]. Thus, a low Gini coefficient indicates more equal income or wealth distribution, while a high Gini coefficient indicates more unequal distribution. Zero corresponds to perfect equality (everyone having exactly the same income) and 1 corresponds to perfect inequality (where one person has all the income, while everyone else has zero income).

In 1968, the sociologist Robert Merton used the term "Matthew Effect" to describe the phenomenon that the rich get richer and the poor get poorer [21]. The "Matthew Effect" is also found in some areas of life such as wealth, achievement, fame, success etc [21–24]. The Matthew Effect for Countries (MEC) was also discovered [22].

In this paper, we study wealth distribution by using evolutionary games on different networks. The simulation results show the Pareto power law distribution for the wealthy population and the Gibbs or log-normal distribution for the low income group. We suggest that the social networks for the high income population and the low income group are different. The dependence of the Gini coefficient on the game parameters is also investigated.

2. The model

Previous studies of wealth distribution often adopted a kinetic exchange model in which each agent is a gas molecule and each trading is a money-conserving collision [9–11,27–29]. One can refer to [10,11] for a detailed review of historical data, empirical analysis and models of wealth distribution. These models approximate well a steady economy. However, the total wealth of the system is reserved and will not vary with time [9,12].

The evolutionary games theory has been widely used to characterize some social and biological processes [30–38]. In a typical Prisoner's Dilemma (PD) or Snowdrift Game (SG), two players simultaneously decide whether to cooperate (C) or defect (D). Each player will get a payoff in each step and then the players will choose to change or keep their strategy on the basis of some learning strategies. One can see that both games are intrinsically suitable for characterizing economic activities such as cooperation, decision, payoff and wealth accumulation [25,26].

Our simulation starts from establishing the underlying cooperation network structure. We consider two different social networks in this paper: the Erdos–Renyi random network and the scale-free network. Starting with N disconnected nodes, the ER random graphs are generated by connecting couples of randomly selected nodes, prohibiting multiple connections, until the number of edges equals $L_{\max}$. The scale-free social network is constructed according to the Barabási–Albert (BA) scale-free network model [2] with the "growth" and "preferential attachment" mechanisms. The BA model reproduces well the power law degree distribution which is in good agreement with the empirical evidence. In this model, starting from m_0 fully connected vertices, one vertex with $m \leq m_0$ edges is attached at each time step in such a way that the probability Π_i of being connected to the existing vertex i is proportional to the degree k_i of the vertex, i.e. $\Pi_i = \frac{k_i}{\Sigma_j k_j}$, where j runs over all existing vertices. In our simulation, we set $N = 10000$ for both kinds of graphs. And we set $m_0 = m = 5$ for BA networks, $L_{\max} = 49985$ for an ER random network, so that all networks have the same density of links.

In the PD or SG, each player can either 'cooperate' (invest in a common good) or 'defect' (exploit the other's investment). Initially, equal percentages of cooperators or defectors were randomly distributed among the agents (vertices) of the population. At each time step, the agents play the game with their neighbors and get payoffs according to the game rules. In the PD, a defector exploiting a cooperator gets an amount T and the exploited cooperator receives S. Two players both receive R upon mutual cooperation and P upon mutual defection, such that $T > R > P > S$. Thus in a single play of the game, each player should defect [35]. In the Snowdrift Game (SG), the order of P and S is exchanged, such that $T > R > S > P$ and thus SG is more in favor of cooperation. We rescale the games such that each depends on a single parameter [33,34]. For the PD, we choose the payoffs to have the values $T = b > 1$, $R = 1$, and $P = S = 0$, where the only parameter, $1 \leq b \leq 2$, represents the advantage of defectors over cooperators. This selection of values preserves the essentials of the evolutionary Prisoners' Dilemma. None of our findings will be qualitatively changed if one sets P to a value that is positive but significantly below 1.0 (so that $T > R > P > S$ is strictly satisfied). For the SG, we make $T = 1 + \beta$, $R = 1$, $S = 1 - \beta$, and $P = 0$ with $0 \leq \beta \leq 1$ as the only parameter.

The evolution or learning strategy is carried out by implementing the finite population analogue of replicator dynamics [30,34]. In each step, all pairs of directly connected individuals x and y engage in a single round of a given game. The total payoff of agent i for the step is stored as P_i and the accumulative payoff (wealth) of agent i since the beginning of the simulation is stored as W_i. Then the strategy of each agent (cooperate or defect) is updated in parallel according to the richest-following rule: whenever a site x is updated, a neighbor y with the most payoff P_y in this time step is drawn among all its k_x neighbors (where k_x is the connectivity or degree of site x), and then the site x will copy the strategy of the chosen neighbor y. This mechanism is adopted to reflect the common practice of agents in the economy: that they will probably learn from richest neighbors.