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ScienceDirect
Physica A 371 (2006) 112–117
PHYSICA A
www.elsevier.com/locate/physa

Living in an irrational society: Wealth distribution with correlations between risk and expected profits

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Available online 17 May 2006

Abstract

Different models to study the wealth distribution in an artificial society have considered a transactional dynamics as the driving force. Those models include a risk aversion factor, but also a finite probability of favoring the poorer agent in a transaction. Here, we study the case where the partners in the transaction have a previous knowledge of the winning probability and adjust their risk aversion taking this information into consideration. The results indicate that a relatively equalitarian society is obtained when the agents risk in direct proportion to their winning probabilities. However, it is the opposite case that delivers wealth distribution curves and Gini indices closer to empirical data. This indicates that, at least for this very simple model, either agents have no knowledge of their winning probabilities, either they exhibit an "irrational" behavior risking more than reasonable.

Keywords: Econophysics; Wealth distribution; Pareto's law

A study of the distribution of the income of workers, companies and countries was presented, more than a century ago, by Italian economist Vilfredo Pareto. He investigated data of income for different European countries and found a power-law distribution that seems to be independent on particular economic condition of each country. He found [1] that the distribution of income and wealth follows a power-law behavior where the cumulative probability $P(w)$ of people whose income is at least w is given by $P(w) \propto w^{-\alpha}$, where the exponent α is named today Pareto index. The exponent α for several countries was $1.2 \leqslant \alpha \leqslant 1.9$. However, recent data indicate that, even though Pareto's law provides a good fit to the distribution of the high range of income, it does not agree with observed data over the middle and low range of income. For instance, data from Japan [2], the United States of America and the United Kingdom [3–5] are fitted by a lognormal or Gibbs distribution with a maximum in the middle range plus a power law for the highest income. The existence of these two regimes may be qualitatively justified by stating that in the low and middle income classes the

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0378-4371/$ - see front matter
doi:10.1016/j.physa.2006.04.088

process of accumulation of wealth is additive, causing a Gaussian-like distribution, while in the high-income class the wealth grows in a multiplicative way, generating the power-law tail.

Different models of capital exchange among economic agents have been recently proposed. Most of these models consider an ensemble of interacting economic agents that exchange a fixed or random amount of a quantity called ''wealth''. In the model of Dragulescu and Yakovenko [3,6] this parameter is associated with the amount of money a person has available to exchange, i.e., a kind of economic ''energy'' that may be exchanged by the agents in a random way. The resulting wealth distribution is a Gibbs exponential distribution, as it would be expected. An exponential distribution as a function of the square of the wealth is also obtained in an extremal dynamics model where some action is taken, at each time step, on the poorest agent, trying to improve its economic state [7,8]. In the case of this last model a poverty line with finite wealth is also obtained, describing a way to diminish inequalities in the distribution of wealth [9]. In order to try to obtain the power-law tail several methods have been proposed. Keeping the constraint of wealth conservation a detailed studied proposition is that each agent saves a fraction—constant or random—of their resources [6]. One possible result of those models is condensation, i.e., the concentration of all the available wealth in just one or a few agents. To overcome this situation different rules of interaction have been applied, for example, increasing the probability of favoring the poorer agent in a transaction [8,10–12], or introducing a cut-off that separates interactions between agents below and above a threshold [13]. Most of these models are able to obtain a power-law regime for the high-income class, but for a limited range of the parameters, while for the low income, the regime can be approximately fitted by an exponential or lognormal function. However, in all those models the risk-aversion (or saving propensity) of the agents is determined at random with no correlation with the probability of winning in a given interaction. Also, possible correlations between wealth and probability of interaction are not considered.

Here, we assume that the agents have some previous knowledge of their winning probability and they adjust their risk-aversion factor in correlation with this winning probability. As in previous models, we consider a population of $N = 10^5$ interacting agents characterized by a wealth w_i and a risk aversion factor β_i. We chose as initial condition for w_i a uniform distribution between 0 and 1000 arbitrary units. For each agent i, the number $[1-\beta_i]$ measures the percentage of wealth he is willing to risk. At each time step t we select at random the two agents i and j that will exchange resources. Then, we set the quantity to be exchanged between these two agents as the minimum of the available resources of both agents, i.e. $\mathrm{d}w = \min[(1-\beta_i)w_i(t); (1-\beta_j)w_j(t)]$. Finally, following previous works we consider a probability $p \geqslant 0.5$ of favoring the poorer of the two partners [10,11],

$$p = \frac{1}{2} + f \times \frac{|w_i(t) - w_j(t)|}{w_i(t) + w_j(t)}, \tag{1}$$

where f is a factor going from 0 (equal probability for both agents) to $\frac{1}{2}$ (highest probability of favoring the poorer agent). Thus, in each interaction the poorer agent has probability p of earn a quantity $\mathrm{d}w$, whereas the richer one has probability $1-p$. Now we consider that in each transaction both participants know this probability and adjust their risk-aversion β according to the value of p. If the agents are ''rational'' they will risk more when they have a higher probability of winning so, taking into account that p varies between 0.5 and 1, we first consider that in each interaction:

$$\begin{aligned} \beta_{rich} &= 2\alpha_r(p - 0.5), \\ \beta_{poor} &= 2\alpha_p(1 - p), \end{aligned} \tag{2}$$

with α_r and α_p ranging from 0 to 1. This correlation between the risk-aversion and p is plotted in Fig. 1, where we change α_r and α_p in order to display the possible variations of the rich and poor tactics, starting with a risk-aversion given by $\alpha_r = \alpha_p = 1$ and then decreasing the slope from 2 to 0 (so decreasing α's from 1.0 to 0.0) for the richer agent (Fig. 1, left panel) or for the poorer agent (Fig. 1, right panel) upto arriving to a constant risk-aversion equal to zero.

In Fig. 2 we have plotted the wealth distribution corresponding for changing strategies of the poorer agent. We have not represented the case when it is the richer agent strategy that changes because we observe that in this case the wealth distribution is independent of the changes and is always equal to the curve (a) of Fig. 2.