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Simulating the wealth distribution with a Richest-Following strategy on scale-free network

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Abstract

In this paper, we investigate the wealth distribution with agents playing evolutionary games on a scale-free social network adopting the Richest-Following strategy. Pareto's power-law distribution (1897) of wealth is demonstrated with power factor in agreement with that of US or Japan. Moreover, the agent's personal wealth is proportional to its number of contacts (connectivity), and this leads to the phenomenon that the rich gets richer and the poor gets relatively poorer, which agrees with the Matthew Effect.

Keywords: Wealth distribution; Game theory; Scale-free network; Matthew Effect

1. Introduction

Complex networks can describe a wide range of systems of high importance, ranging from nature to society and biological systems. Since the pioneer work by Watts and Strogatz [1] with the discovery of small-world behavior and by Barabási and Albert [2] with the scale-free property, complex networks has attracted continuous attention from the physics community [3]. Prototypical examples of complex networks covers as diverse as protein–protein interaction networks, metabolic networks, the Internet, the World Wide Web, airport networks and many types of collaborative communities in society. By representing the agents of a given population with vertices, and the contacts between agents with edges, network theory provides a natural framework to describe the population structure [4]. One can easily found that well-mixed populations can be represented by complete (fully-connected, regular) networks and spatially-structured populations can be associated with regular networks. Recently, much empirical evidence of real social networks has revealed that they are associated with a scale-free, power-law degree distribution, $d(k) \sim k^{-\gamma}$ with $\gamma_{actor} = 2.3 \pm 0.1$ for movie actor collaboration network [5], $\gamma_{science} = 2.1$ and 2.5 for science collaboration graph [6], $\gamma_f = 3.5 \pm 0.2$ and $\gamma_m = 3.3 \pm 0.2$ for females and males in human sexual contacts [7], etc.

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It is well known that even in developed countries, it is common that 90% of the total wealth is owned by only 5% of the population. The distribution of wealth is often described by 'Pareto'-tails (1897), which decay as a power law of large wealths [8]:

$$P(W) \propto W^{-(1+v)}, \tag{1}$$

where $P(W)$ is the probability of finding an agent with wealth greater than W, and the value of v was found to lie between 1 and 2 for both individual wealth and company sizes [9–13]. Studies on real data show that the high-income group indeed follows the Pareto law, with v varying from 1.6 for USA [9] to 1.8–2.2 in Japan [10].

Another interesting phenomenon in personal wealth is the 'Matthew Effect'. The 'Matthew Effect' refers to the idea that in some areas of life (wealth, achievement, fame, success, etc.), the rich gets richer and the poor gets poorer [14–17]. In the year of 1988, the sociologist Robert Merton used the term 'Matthew Effect' to describe the phenomenon of scientists giving exclusive credit to the most distinguished one among several equally deserving candidates [14]. The Matthew Effect for Countries (MEC) was also discovered [15].

In this paper, we adopt the scale-free network to represent the cooperative structure in population and study the wealth increment by using evolutionary games as a paradigm for economic activities and simulate the personal wealth distribution in society. The evolutionary games theory has been widely used to characterize some social and biological processes, such as the cooperative behavior in systems consisting of selfish individuals [18–27]. In Ref. [20], Szabo and Fath present a nice review about evolutionary games on graphs. In the typical Prisoner's Dilemma (PD) and Snowdrift Game (SG), two players simultaneously decide whether to cooperate (C) or defect (D). Each player will get a payoff in each step and then the players will choose to change their strategy or to keep their strategy unchanged based on some take-over strategies. One can see that both games' dynamics are intrinsically suitable for characterizing the payoff and wealth accumulating behavior in economy.

Different from our previous work [28], we investigate the wealth problem with using the Richest-Following strategy. Richest-Following is a common action of people in economy that they usually follow the strategy of a specific neighbor who gain most profit. The simulation results show the Pareto wealth distribution along with the Matthew Effect in economy.

2. The model

The previous studies of wealth distribution often adopt an ideal-gas model in which each agent is represented by a gas molecule and each trading is a money-conserving collision [29–33]. One can refer to [29] for a detailed account of historical data, empirical analyses and models of wealth distribution. These conserved wealth models well approximate a steady economy: the total wealth growth rate is much slower than the frequency of trading/exchanging activity. And hence the conserved wealth models are very good approximations at short time scales. However, these ideal-gas models can only reproduce the Gibb distribution or Gaussian-like stationary distribution of money [30] and they are not suitable for studying the material wealth distribution because, in general, the total material wealth of the system will increase with time [31,33].

Our simulation starts from establishing the underlying cooperation network structure according to the Barabási–Albert (BA) scale-free network model [2], since the classic regular or random networks are not good representations of many real social networks. With the 'growth' and 'preferential attachment' mechanisms, the model well reproduces power-law degree distribution which is in good agreement with the empirical evidence. In this model, starting from m_0 fully connected vertices, one vertex with $m \leqslant m_0$ edges is attached at each time step in such a way that the probability Π_i of being connected to the existing vertex i is proportional to the degree k_i of the vertex, i.e., $\Pi_i = k_i/\Sigma_j k_j$, where j runs over all existing vertices. In our simulation, we set $m_0 = m = 2$.

In the PD or SG, each player can either 'cooperate' (invest in a common good) or 'defect' (exploit the other's investment). Initially, an equal percentage of cooperators or defectors was randomly distributed among the agents (vertices) of the population. At each time step, the agents play the game with their neighbors and get payoff according to the game rules. In the PD, a defector exploiting a cooperator gets an amount T