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Degree and wealth distribution in a network induced by wealth

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Abstract

A network induced by wealth is a social network model in which wealth induces individuals to participate as nodes, and every node in the network produces and accumulates wealth utilizing its links. More specifically, at every time step a new node is added to the network, and a link is created between one of the existing nodes and the new node. Innate wealth-producing ability is randomly assigned to every new node, and the node to be connected to the new node is chosen randomly, with odds proportional to the accumulated wealth of each existing node. Analyzing this network using the mean value and continuous flow approaches, we derive a relation between the conditional expectations of the degree and the accumulated wealth of each node. From this relation, we show that the degree distribution of the network induced by wealth is scale-free. We also show that the wealth distribution has a power-law tail and satisfies the 80/20 rule. We also show that, over the whole range, the cumulative wealth distribution exhibits the same topological characteristics as the wealth distributions of several networks based on the Bouchaud–Mèzard model, even though the mechanism for producing wealth is quite different in our model. Further, we show that the cumulative wealth distribution for the poor and middle class seems likely to follow by a log-normal distribution, while for the richest, the cumulative wealth distribution has a power-law behavior.

Keywords: Wealth distribution; Degree distribution; Network model

1. Introduction

The concept of a network is considered to be an efficient frame for understanding a variety of complicated systems in the real world. These include biological phenomena, such as chemical reactions in a cell [1–3], social interactions accompanied with information (or knowledge) relations [4–7], and social structures designed to distribute natural resources, social information, and physical products [8–11]. Networks are also useful in explaining and handling the complexity of such systems. A number of network models have been proposed over the last few decades. The random network model [12], the small world network model [13], and the

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scale-free network model [14] are prominent. The scale-free (SF) network model has been widely used in recent works to explain some interesting network phenomena observed in the real world, such as the small world phenomenon. The SF network model is based on the key concept of preferential attachment, which is governed by a topological (or structural) quantity. Preferential attachment endows a growing network with heterogeneity in its structure, such as a power-law degree distribution.

In examining real social networks, we often find that they fit the SF model for reasons other than preferential attachment. In some cases, it might be quite unnatural to use a structurally defined concept such as the degrees of the nodes representing individuals. Instead, the evolution of some networks seems to depend on non-structural properties such as friendship, culture, and wealth. For example, in the fitness model [15,16], two nodes are connected when they create mutual benefits through interactions. The correspondence of such a network to the SF model can be explained by vertex-intrinsic fitness, which measures the importance of each vertex, instead of by preferential attachment. The fitness model puts a more active interpretation on the meanings of the nodes and links in a network, in the sense that every node participates in a network to derive a profit by means of links, each of which provides an interaction between two nodes. This certainly seems to be a more natural and reasonable view of social networks. The fitness model, however, lacks some fundamental and essential details, such as motives for individuals to become nodes in a network, the real targets of the activities of nodes, and procedures for creating benefits.

Let us consider the evolution of the social structure and the wealth distribution of a society, in terms of the growth of wealth. Wealth is the main target of the economic activities of individuals and corporations, and is also one of the most powerful motives for human relationships in a society. Therefore, information regarding wealth can be more important than topological information, such as degrees, paths, and clusters, in providing a direct explanation of the economic activities of individuals in a society. But topological information remains meaningful, since individuals gain wealth through the structural features of the society. To some extent, individuals to join a society produce and accumulate wealth through social activities that exploit the structure of the society. Using a society as a network, wealth, individuals, and the structure of the society evolve reciprocally.

Macroscopically, the structure of a society influences on individual's activities in producing and accumulating wealth, and microscopically these same individuals continue to try to acquire more wealth. This suggests a need for a network model that focuses on the interactions between wealth and the structure of a society. There has already been quite a lot of research on economic models of wealth distribution [17–20] and, more relevantly, on the social networks on which an economic model runs [21–23,25,26]. Coelho et al. [26] proposed a model in which the network structure is generated in parallel with the accumulation of wealth through some dynamic rules. In this model, the growing network converges to a stable topology, and the wealth distribution function has a power-law tail for the richest class in the model; but links make no contribution to wealth accumulation, nor is wealth creation linked to the innate ability of each node.

In this paper, we propose a new network model which follows on from this discussion. We are going to assume that there exists a common motive that makes individuals become nodes in the network, and we shall call this wealth. We will also assume that, once an individual joins the network, they try to produce wealth using their intrinsic abilities and the structure of the network. In our model, wealth-producing ability of the node in the network is distributed between 0 and 1 and each link provides an opportunity for the nodes that it connects to produce new wealth. From an analysis of this network, we obtain an explicit relation between wealth and the structure of the network, in the form of an integral equation which can be expressed in terms of the conditional expectation of the degree and the conditional expectation of the accumulated wealth of each node. This equation provides a theoretical basis for explaining the feedback relationship between the nodes and the network. By solving the integral equation, we show that the degree distribution is scale-free and that the wealth distribution has a power-law tail. Further, the numerical results of simulations show that the cumulative wealth distribution satisfies the so-called '$\frac{80}{20}$ rule', whereby the richest 20% of nodes hold about 80% of the wealth, which is a representative property observed in Pareto regimes.

We also show that, over the whole range, the cumulative wealth distribution (the probability that the wealth of an individual is greater than a given value) has topologically the same behavior as that observed in the cumulative wealth distributions of several networks [23,24] that use the Bouchaud–Mèzard (BM) model [17], even though the way in which wealth is produced in our model is quite different from the mechanism which derives the BM model. Moreover, we also show that the cumulative wealth distribution for the poor and