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An analytic treatment of the Gibbs–Pareto behavior in wealth distribution

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Abstract

We develop a general framework, based on Boltzmann transport theory, to analyze the distribution of wealth in societies. Within this framework we derive the distribution function of wealth by using a two-party trading model for the poor people, while for the rich people a new model is proposed, where interaction with wealthy entities (huge reservoir) is relevant. At equilibrium, the interaction with wealthy entities gives a power-law (Pareto-like) behavior in the wealth distribution, while the two-party interaction gives a Boltzmann–Gibbs distribution.

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Keywords: Econophysics; Wealth distribution; Pareto law; Boltzmann–Gibbs distribution

1. Introduction

Inequality in the distribution of wealth in the population of a nation has provoked a lot of studies. It is important for both economists and physicists to understand the root cause on this inequality: whether stochasticity or a loaded dice is the main culprit for such a lop-sided distribution. While it has been empirically observed by

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Pareto [1] that the higher wealth group distribution has a power-law tail with exponent varying between 2 and 3, the lower wealth group distribution is exponential or Boltzmann–Gibb's like [2,3]. The Boltzmann–Gibb's law has been shown to be obtainable when trading between two people, in the absence of any savings, is totally random [4–6]. The constant finite savings case has been studied earlier numerically by Chakraborti and Chakrabarti [4] and later analytically by us [5]. As regards the fat tail in the wealth distribution, several researchers have obtained Pareto-like behavior using approaches such as random savings [7,8], inelastic scattering [9], generalized Lotka Volterra dynamics [10], analogy with directed polymers in random media [11], and three-parameter-based trade–investment model [12].

In this paper, we try to identify the processes that lead to the wealth distribution in societies. Our model involves two types of trading processes—tiny and gross. The tiny process involves trading between two individuals, while the gross one involves trading between an individual and the gross system. The philosophy is that small wealth distribution is governed by two-party trading, while the large wealth distribution involves big players interacting with the gross system. The poor are mainly involved in trading with other poor individuals, whereas the big players mainly interact with large entities/organizations such as government(s), markets of nations, etc. These large entities/organizations are treated as making up the gross system in our model. The gross system is thus a huge reservoir of wealth. Hence, our model invokes the tiny channel at small wealths while at large wealths the gross channel gets turned on.

2. General framework

We will now develop a formalism similar to the Boltzmann transport theory so as to obtain the distribution function $f(y, \dot{y}, t)$ for wealth y, net income $\dot{y}$ (or total income after consumption) as a function of time t. Similar to Boltzmann's postulate, we also postulate a dynamic law of the form

$$\frac{\partial f}{\partial t} = \left\{\frac{\partial f}{\partial t}\right\}_{\text{ext source}} + \left\{\frac{\partial f}{\partial t}\right\}_{\text{interaction}} . \tag{1}$$

The first term on the right-hand side (RHS) describes the evolution due to external income sources only, while the second one represents contribution from entirely internal interactions.

2.1. Model for tiny trading

Individuals, possessing wealth smaller than a cutoff wealth y_c, engage in two-party trading, where two individuals, 1 and 2, put forth a fraction of their wealth $(1-\lambda_t)y_1$ and $(1-\lambda_t)y_2$, respectively [with $0 \leqslant \lambda_t < 1$]. Then the total money $(1-\lambda_t)(y_1+y_2)$ is randomly distributed between the two. The total money between the two is conserved in the two-party trading process. We assume that probability of trading by