Available online at www.sciencedirect.com
SCIENCE DIRECT®
ELSEVIER
Int. J. Production Economics 93–94 (2005) 357–373
international journal of production economics
www.elsevier.com/locate/dsw

Turnpike policies for periodic review inventory model with emergency orders

Stanisław Bylka*

Institute of Computer Science, Polish Academy of Sciences, ul. Ordona 21, 01-237 Warsaw, Poland

Abstract

This paper presents a periodic review capacitated lot sizing model with limited backlogging and a possibility of emergency orders. The main intention of placing emergency orders is to satisfy the demand as soon as a shortage occurs. It is assumed that the demands are independently distributed in successive periods. There are two resupply modes available: a regular mode and an emergency mode. Hence, no lead-time of emergence orders is assumed, the purchase price could be high. The measure of effectiveness is the total (or average per period) expected cost, which includes holding cost, shortage cost and both types of order costs.

The minimum cost is obtained by considering this system as a discrete-time Markov decision process. We use this model to describe a simple and efficient value function algorithm for finding optimal policies. We find propositions on specially structured optimal policies generalizing classical results on (s, S) policies. Some connections of the turnpike policies and optimal infinite policies are presented. Computational results are given through numerical examples.

Keywords: Inventory models; (s, S) policies; Turnpike policies; Emergency orders

1. Introduction

The classic lot size model involves the uncapacitated production or order of a single product and the storage in a warehouse of unlimited capacity. With respect to unsatisfied demands, it is either the full lost-sales or the full-backlog model. Various modifications have been made to this classic model. In some models, warehouse space puts a constraint on the physical inventory. Some of the other include the introduction of two types of orders and some bounds on production (on size of the orders). We present a Markov model of a single firm which look for inventory policy to procure a commodity with two kinds of supply modes (or two alternative suppliers): one a mode with a lag of deliveries, the other an express

*Tel.: +48-22-36-28-41; fax: +48-22-37-65-64.
E-mail address: bylka@wars.ipipan.waw.pl (S. Bylka).

0925-5273/$ - see front matter
doi:10.1016/j.ijpe.2004.06.033

shipment without lag but at a higher price (see Arrow et al., 1958, in 5. Deliveries). We call the orders on the first mode emergency (or express) orders, while those of the other type are regular orders. We assume there is the lead time equal to 1 for possible regular orders. The main intention of placing emergency orders is to reduce the delay in meeting the demand. However its high purchase price will be balanced against high shortage costs.

There are quite a few articles dealing with emergency orders. Through the incorporation of such lead times and order bands, our model complements the models Scheller-Wolf and Tayur (1998). Periodic review inventory systems in which at each review time point, the inventory manager must chose between alternative supply modes and then order up to particular level have been examined in Chiang and Gutierrez (1998). Tijms (1994) uses the theory of Markov decision processes to prove the optimality of (s, S)-policies for inventory problems with stochastic demands. In particular, he imposes upper and lower bounds on the inventory positions. This provides a finite action space, as well as bounded costs. Under similar conditions we look for optimal policies in a more general setting. We show that for finite time horizon problems it is optimal for the manager to follow some modified (s, S) policies.

The classical economic order quantity (EOQ) formula is perhaps the best known decision formula in the production inventory literature. We present a natural generalization of the EOQ formula as the quantity chosen by a turnpike policy in the model with partial backlogging and stockouts. We show that the decision given by turnpike policies can be used in the rolling horizontal procedure with a finite horizon. Moreover, we formulate a simple and efficient algorithm to obtain such policy.

The formulation of the model under study is given in Section 2. In Section 3, the Markov decision processes point of view is presented. We use them to formulate the dynamic programming equations. Algorithms that utilize the optimality conditions are presented in Sections 4 and 5 along with the results on the existence of an optimal feedback policy. Computational results with the algorithms are given through examples and in the tables. Interactions between turnpike policies, optimal infinite inverse policies and rolling horizontal plans are presented in Section 6.

2. Dynamic lot size model with emergency orders

For the purpose of this paper, we introduce the following version of the dynamic one-product inventory model. Time is taken to be discrete and the random demands for different periods are independent identically distributed. We assume, that emergency (or express) orders arrive immediately (so the lead time of a regular order is equal to 1). In each period the sequence of events will be as follows: the regular order (placed in previous to that period) arrives, the emergency or regular orders are placed, the emergency order arrives, a demand occurs and costs are incurred. It is assumed that the shortage is partially backlogged and partially lost. The total (physical) inventory is restricted by warehouse size constraint. We have been working under the assumption that the demands are non-negative integers. Therefore, the other parameters, instead of cost parameters, will be consider as integers. The following terminology and notation are used. For the dynamic parameter of the problem we define:

d_t the demand in period t, $t = 1, 2, \ldots, T$, where $T \leqslant +\infty$.
$\phi(n)$ the probability of the event that $d_t = n$. We assume $\sum_{n=0}^{\infty} \phi(n) = 1$, $\phi(n) = 0$ for $n < 0$ and $\mu = \sum_{n=0}^{\infty} n\phi(n) < \infty$, with the notation: $\bar{\phi}(n) = \sum_{v=n}^{\infty} \phi(v)$.

For possible constraints we use the following parameters:

$\bar{m} \geqslant 0$ the shortage above $\bar{m}$ is lost (stockout). We set $\bar{m} = 0$ and $\bar{m} = +\infty$ in the full-stockout and the full-backlog case, respectively.