



## Cost-effective ordering policies for inventory systems with emergency order

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### ABSTRACT

Most of the studies on inventory control reported in earlier contributions deal with the optimization problems minimizing an expected cost criterion such as the long-run average cost. However, when the plant engineers design the inventory systems in practical situations, they must take account of reliability into the inventory system as well as economical aspect, where reliability can be defined as the probability that the stock is not depleted until a pre-specified time. In this paper, we discuss the inventory control policies that provide a balance between economical and reliability requirements. By applying the cost effectiveness criterion, which simultaneously includes the effects of system availability and expected cost, as optimality one, we derive the optimal inventory replenishment policies of two kinds of inventory models. Finally, with a set of numerical examples, we show that the optimal inventory policies of the models under consideration make the stationary availability increase.

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### 1. Introduction

The importance of inventory control in business has increased dramatically with the development of sophisticated CIM (Computer Integrated Manufacturing) concept. It gives the logic and basis on controlling the manufacturing system as well as plays the role of releasing the surplus operating capital tied up in excessive inventories. Most of the studies on inventory control reported in the literature (see Hadley & Whitin, 1963; Silver, Pyke, & Peterson, 1998) deal with the optimization problems minimizing an expected cost criterion such as the long-run average cost. However, when the plant engineers design the inventory systems in practical situations, they are often required to take account of reliability into the inventory system as well as usual economical aspect, where reliability can be defined as the probability that the stock is not depleted until a pre-specified time.

In inventory systems, the shortages are sometimes caused by a delivery-lag. In the early contributions, Allen and D'Esopo (1968a, 1968b), Barankin (1961), Neuts (1964) and Rosenshine and Obee (1976) address some inventory models to avoid the shortage by an additional order form. Lau and Zhao (1993) and Moinszadeh and Nahmias (1988) analyze the optimal ordering policies for the inventory system with two supply modes, and extended the seminal results by Whittmore and Saunders (1977). In fact, the problem of shortage may be overcome to some extent by splitting orders among a number of suppliers; but such an ordering form is rather costly and is not always realized in practice, even if the stock control system with multiple suppliers is available.

The above works consider the expected cost as optimality criterion. Some researchers deal stochastic inventory control problems with other optimality criterion such as service level criterion which introduces a service level constraint instead of shortage cost (see Chen & Krass, 2001; Cohen, Kleindorfer, & Lee, 1988; Nahmias, 1993). They define service level as the availability of stock in a probabilistic or expected sense. One major disadvantage of this approach is that it is perceived to be less tractable mathematically.

The purpose of this paper is to implement the cost effectiveness criterion (Hunter, 1963) which provides a balance between economical and reliability requirements to the standard inventory models, and to propose the cost-effective inventory policy from reliability as well as cost minimization view points. We consider two kinds of inventory models referred to as Model 1 and Model 2 from onwards. Model 1 is a continuous review inventory model in which the stock level is continuously reviewed until a specified time and the items are replenished by the regular order at that time if no shortage has occurred. If the stock level becomes 0 before the regular ordering time, then the items are replenished by the expedited order. On the other hand, Model 2 is a periodic review inventory model in which the stock level is reviewed only at a specified time and one of the regular and expedited orders is placed at that time. Dohi, Kaio, and Osaki (1995) analyzed Model 1 under long-run average cost and expected total discounted cost criteria. Our objective in this study is to derive optimal inventory policies for Models 1 and 2 maximizing the cost effectiveness criterion. The cost effectiveness criterion can be regarded as an optimality criterion which assesses the effective system operating time in one cycle relative to the mean operating costs in that cycle. It is to be mentioned here that the inventory models under consideration have the same mathematical structure as

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the order-replacement models in the context of reliability theory (see Kaio & Osaki, 1977; Osaki, 1992; Osaki, Kaio, & Yamada, 1981; Thomas & Osaki, 1978a, 1978b).

The paper is organized as follows: Section 2 describes the nomenclature, notation and assumptions. In Sections 2 and 3, the continuous and periodic review inventory models are explained, cost effectiveness is defined and the optimal ordering times are derived for a fixed order quantity. Assuming a Poisson process demand, we obtain the explicit formulas of cost effectiveness for the two inventory models in Section 5. We also investigate numerically the performance of cost-effective design for the inventory models. Finally, Section 6 concludes the paper.

**2. Nomenclature, notation and assumptions**

**2.1. Nomenclature**

IHR	increasing hazard rate
Inventory level process	a continuous time stochastic process representing the on-hand inventory level which is decreased by the satisfaction of a demand
Inventory policy	the policy which consists of both the regular ordering time and the order quantity

**2.2. Notation**

$\{X(t), t \geq 0\}$	inventory level process
$\{N(t), t \geq 0\}$	cumulative demand process
$Q$	order quantity
$t_0$	ordering time for the regular order
$F(t), f(t)$	cdf and pdf for the time to first shortage
$\bar{F}(\cdot) \equiv 1 - F(\cdot)$	survivor function
$r(t) \equiv f(t)/\bar{F}(t)$	hazard rate for $F(t)$
$R(t) \equiv \{F(t + L_2) - F(t)\}/\bar{F}(t)$	conditional hazard function during interval $(t, t + L_2]$
$i$	model indicator, that is, $i = 1, 2$ correspond to Model 1 and Model 2, respectively
$L_1, L_2$	constant lead times for the expedited and regular orders, respectively
$c_1, c_2$	cost per unit amount for the expedited and regular orders, respectively
$h$	inventory holding cost per unit amount per unit time
$k$	penalty cost per unit time suffered for the shortage period
$V_i(t_0, Q)$	expected cost for one cycle
$T_i(t_0, Q)$	mean time for one cycle
$M(Q)$	mean effective time for one cycle
$A_i(t_0, Q)$	stationary availability
$Z_i(t_0, Q) = M(Q)/V_i(t_0, Q)$	cost effectiveness
$\text{gamf}(\cdot)$	upper incomplete gamma function

**2.3. Assumptions**

(A-1) The inventory management begins operating at time 0, and the planning horizon is infinite.

- (A-2) Without loss of generality, the inventory level is initially set to  $Q(>0)$ .
- (A-3) Shortages, if occurred, are not backlogged.
- (A-4) The time period from time 0 to the time when the inventory level becomes  $Q$  next is one cycle and the same cycle repeats itself continually.
- (A-5) The sum of the ordering and the shortage costs by the expedited order is greater than the sum of those by the regular order, i.e.,  $c_1Q + kL_1 > c_2Q + kL_2$ .

**3. Continuous review model (Model 1)**

**3.1. Model description**

If the stock is depleted up to a pre-specified time  $t_0 \in (0, \infty)$ , the emergency order is placed immediately at that time and after a lead time  $L_1(> 0)$ ,  $Q$  units are replenished. We call  $t_0$  as ordering time, in this paper. On the other hand, if the stock is not depleted before the time  $t_0$ , the regular order is made at the time  $t_0$ , and  $Q$  units are replenished after a lead time  $L_2(> 0)$ .

We suppose that the time when the inventory level becomes 0 for the first time obeys a distribution  $F(t) = F(t; Q)$  with a density function  $f(t) = f(t; Q)$ . Of course, specifying the inventory level process  $X(t)$  leads to characterize  $F(t)$  and  $f(t)$ . We define the first passage time as follows:

$$\tau \equiv \inf\{t \geq 0; X(t) = 0 | X(0) = Q\}. \tag{1}$$

We define the indicator function  $I_{\{A\}}$  for the event  $\{A\}$ . Fig. 1 shows the schematic illustration of the inventory model under consideration where the events  $\{A1\}$ ,  $\{A2\}$  and  $\{A3\}$  are such that

$$X(t)I_{\{A1\}} \equiv \{X(t); \min_{0 \leq t \leq t_0} X(t) = 0\}, \tag{2}$$

$$X(t)I_{\{A2\}} \equiv \{X(t); \min_{0 \leq t \leq t_0} X(t) > 0 \text{ and } \min_{0 \leq t \leq t_0 + L_2} X(t) = 0\}, \tag{3}$$

$$X(t)I_{\{A3\}} \equiv \{X(t); \min_{0 \leq t \leq t_0 + L_2} X(t) > 0\}, \tag{4}$$

respectively.

As the inventory process has a regenerative point where the same state repeats periodically, we use renewal reward theory (Ross, 1970) to find the expected cost per unit time which is given by

$$C_1(t_0, Q) = V_1(t_0, Q)/T_1(t_0, Q), \tag{5}$$

where  $V_1(t_0, Q)$ , the expected cost in one cycle is

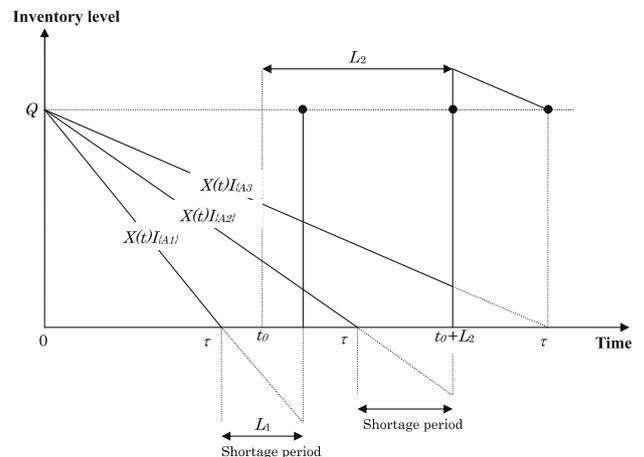


Fig. 1. State diagram for one cycle in Model 1.

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