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Strategic behavior and partial cost sharing

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Abstract

The main objects here are games in which players mainly compete but nonetheless collaborate on some subsidiary activities. Play assumes a two-stage nature in that first-stage moves presume coordination of some subsequent tasks. Specifically, we consider instances where second-stage coordination amounts to partial cost sharing, anticipated and sustained as a core solution. Examples include regional Cournot oligopolies with joint transportation. We define and characterize equilibria, and inquire about their existence.

JEL classification: C71; C72

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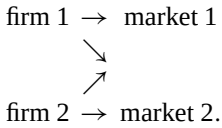
1. Introduction

For motivation consider two firms which compete in as many markets, supplying these with one—or maybe several—homogeneous commodities:



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The said firms interact over *two* stages. *First*, each decides independently how much to produce and bring to every market. *Second*, having produced their quantities, there are gains to be had in coordinating the subsequent transportation from factories to markets. In fact, cost reductions obtain if each firm, fully or partly, serves the nearest market on behalf of his rival.

Similar examples include: coordinated distribution of competing newspapers, or a common shuttle bus serving rival air companies. More generally, one may think of "noncooperative" producers who maintain shared inventories, or organize internal exchange of scarce resources, or outsource some subsidiary tasks jointly. We ask: *ex post, may such secondary activities be coordinated to mutual advantage*? *If so, is it possible to share the associated costs fairly*? *And ex ante, if players anticipate the subsequent cost sharing, can they reach an overall equilibrium*?

Indeed, they can. Under broad and natural assumptions all these questions have positive and intimately related answers. For illustration Section 2 elaborates on the above figure so as to have a running example. Section 3 builds a rather general model, including the duopoly already depicted, and going well beyond it. It is defined there what is meant by an equilibrium.

Since second-stage collaboration constitutes a key part of the overall setting, Section 4 digresses to study *transferable-cost cooperative games*. All our instances concern *cost sharing*, and they fit the form of so-called *production games* (*or production economies*) in which technologies, tasks and endowments are pooled (Dubey and Shapley, 1984; Granot, 1986; Kalai and Zemel, 1982a, 1982b; Samet and Zemel, 1994; Shapley and Shubik, 1969, 1972; Sondermann, 1974).

Section 5 brings out some simple, novel properties of cost-sharing games, extending the results of Owen (1975) to nonlinear instances; see also (Evstigneev and Flåm, 2001) and (Sandsmark, 1999). One desirable property is that *infimal convolution* of convex cost functions yields a nonempty *core*. Another, more useful and practical property is that Lagrange multipliers constitute a (shadow) *price regime* that decentralizes cooperative planning and defines a core imputation. Using such prices, each agent is charged, at the second stage, for his "quantity" less a competitive profit, computed as though he were a price-taker.

Section 6 concludes by briefly mentioning how equilibrium could be learned or approached.

2. A regional oligopoly

As running example consider *a regional oligopoly*—already motivated in the introduction. Finitely many firms $i \in I$ produce the same homogenous good[1] to be shipped from *origins* $o \in O$ to *destinations* $d \in D$. Both sets O, D are finite and—without loss—regarded as disjoint. Denote by quantity q_{io} the output of firm i at o, and let q_{id} be how much it delivers at d. We tacitly assume that $\sum_{o\in O} q_{io} = \sum_{d\in D} q_{id}$ for every i. The non-

[1] More than one good could easily be accommodated—at the expense of more complex notations.