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Self-organization of hexagonal agglomeration patterns in new economic geography models

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A B S T R A C T

Self-organization of agglomeration patterns for economic models in a two-dimensional economic space is studied from a multi-disciplinary viewpoint of new economic geography, central place theory, and bifurcation theory. Emergence of hexagonal distributions of various sizes in a homogeneous space is predicted theoretically for core–periphery models. The existence of hexagonal distributions as stable equilibria is demonstrated by a comparative static analysis with respect to transport costs for specific core–periphery models. These distributions are the ones envisaged by central place theory and also inferred to emerge by Krugman (1996) for a core–periphery model in two dimensions.

1. Introduction

The evolution of economic agglomeration in cities is a vital factor of economic growth, and its study is an important topic in economic geography.[1] An accepted scenario of this agglomeration is the self-organization of a few large cities from evenly spread economic activities associated with the progress of transportation technology, trade liberalization, and economic integration.

A question to be answered is, "*Where* and *how* is spatial agglomeration self-organized?" Where agglomeration occurs was first studied in central place theory by Christaller (1933), who envisaged the self-organization of market areas of various sizes in a two-dimensional space. *How* self-organization takes place was elucidated not by Christaller's study, but by a study

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[1] See, e.g., Clarke and Wilson (1985) and Munz and Weidlich (1990) for early studies of self-organizing patterns in geography and regional science.

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of Krugman (1991). He introduced the Dixit–Stiglitz model (1977) of monopolistic competition into spatial economics, and modeled product market imperfections occurring in conjunction with increasing returns, transportation costs, and factor mobility. Thereafter, this model blossomed into new economic geography, which is acknowledged as an important branch of international, regional, and urban economics. The modeling, however, remained one-dimensional and the problem of *where* agglomeration occurs is yet to be fully answered. The objective of this paper is to answer *where* and *how* economic agglomeration takes place in a two-dimensional space based on a multi-disciplinary viewpoint of new economic geography, central place theory, and bifurcation theory.

In central place theory of economic geography (Appendix A), Christaller (1933) envisaged self-organization of hexagonal market areas of various sizes in an infinite uniform space in two steps: (1) formation of hexagons of a single size for a single industry and (2) that of overlapping hexagons of various sizes for multi-industries. He demonstrated the dominance of such self-organization in determining the distribution of central places in southern Germany, and his study contributed to empirical investigations of several places, such as Snohomish County (Washington), Southwestern Iowa, Southwestern Ontario, the Niagara Peninsula, and so on (Dicken and Lloyd, 1990, pp.39–43). Sanglier and Allen (1989) used a dynamic model based on central place theory and successfully calibrated the model with socio-economic data for Belgium, 1970–84.

Although "it (central place theory) is a powerful idea too good for being left as an obscure theory" (Fujita et al., 1999a), this theory is based only on a normative and geometrical approach and is not derived from market equilibrium conditions. To reinforce this theory not only from a geographical standpoint but also from an economic viewpoint, it is necessary to give it the following two underpinnings: (i) a microeconomic mechanism for the location equilibrium and (ii) a two-dimensional economic space.

An early attempt to provide central place theory with a microeconomic foundation was made by Eaton and Lipsey (1975, 1982), and a hexagonal distribution of mobile production factors (e.g., firms and workers) in two dimensions was shown to exist as an economic equilibrium for spatial competition (Eaton and Lipsey, 1975). This, however, remained as a partial equilibrium approach and did not investigate the stability of the equilibrium.

In new economic geography, analytical results for these models have been acquired mostly using simple geometries of two places[2] and sometimes employing the racetrack economy,[3] namely, an economy in which initially identical places spread uniformly around the circumference of a circle. Most studies have dealt with a single industry, whereas a few have developed a microeconomic mechanism for multi-industries (Fujita et al., 1999a; Tabuchi and Thisse, 2011).[4] Yet these studies dealt only with one-dimensional economies and relied on the numerical simulation.

The racetrack economy, which is one-dimensional, can accommodate several patterns, and Krugman (1996, p. 91) inferred the following based on the study of a racetrack economy:

> I have demonstrated the emergence of a regular lattice only for a one-dimensional economy, but I have no doubt that a better mathematician could show that a system of hexagonal market areas will emerge in two dimensions.

The limitation of the one-dimensional economy and the need to extend core–periphery models into two dimensions have come to be acknowledged, as cited by Neary (2001, p. 551): "Perhaps it will prove possible to extend the Dixit-Stiglitz approach to a two-dimensional plain." Stelder (2005) conducted a simulation of agglomeration for cities in Europe using a grid of points. Barker (2012) extended the racetrack geometry to two dimensions, conducted a simulation, and compared it with real cities. Although such naïve simulations can yield some information on agglomeration patterns, it is not possible to overcome several difficulties encountered in a two-dimensional economy, such as a plethora of multiple stable equilibria, and theoretical classification and interpretation of these equilibria (Section 4). A firm theoretical basis to classify these equilibria and a systematic methodology to set forth predominant ones must be established to derive implications for policy proposals.

The unification of central place theory and microeconomic mechanisms, which was first attempted by Eaton and Lipsey (1975, 1982), seems to be in sight. Such unification is important in extending the horizon of new economic geography via cross-fertilization with central place theory. As for the aforementioned two underpinnings for central place theory, the first underpinning of a micro economic mechanism seems to have been satisfactorily constructed in new economic geography, whereas the second underpinning of a two-dimensional economic space remains to be constructed.

A proper choice of a spatial platform is an important issue, and there are several candidates (cf., Golubitsky and Stewart, 2002):

- A continuous two-dimensional space without any discretization is an ideal spatial platform. Yet a continuous version of a core–periphery model needs to be developed, and mathematical analysis of this space with large symmetry would become very complicated.

[2] There is a criticism that economic agglomerations, in reality, would emerge at more than two locations, as was stated by Behrens and Thisse (2007) and was empirically evidenced by Bosker et al. (2010).

[3] Agglomeration patterns of the racetrack economy were observed by Krugman (1993), Fujita et al. (1999b), Picard and Tabuchi (2010), Ikeda et al. (2012a), and Akamatsu et al. (2012).

[4] A one-dimensional continuous segment, termed the long narrow economy, was used by Fujita et al. (1999a), combining a core–periphery model for multiple industries with an urban spatial economy. By comparative static analysis with respect to population size, they demonstrated the emergence of an urban hierarchy. Tabuchi and Thisse (2011) studied the racetrack economy for the multi-industry model to produce Christaller-like spatial patterns.