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A generalized impulse control model of cash management

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Abstract

This paper presents a general model of cash management, viewed as an impulse control problem for a stochastic money flow process. Generalizing classical approaches, we represent this process by a superposition of a Brownian motion and a compound Poisson process, controlled by two-sided target-trigger policies. For phase-type distributions for the upward and downward jumps we determine all pertinent cost functionals explicitly. Moreover, the controlled process is studied in steady state. The closed-form results can be used to determine optimal values for the target and trigger values numerically.

Keywords: Cash management; Brownian motion; Compound Poisson process; Superposition; Optional sampling; Target-trigger control; Cost functionals

1. Introduction

Models of cash management, or money demand, can be divided in two broad categories. The first deals with demand by households, pioneered in the famous Baumol–Tobin model (Baumol, 1952; Tobin, 1956) and extended, among others, by Frenkel and Jovanovic (1980), Bar-Ilan (1990), and Chang (1999). In this approach the money stock is described by a downward drifting flow of expenditures until a certain low level is hit, which will trigger a conversion of financial assets to money in order to raise the cash stock to some target level, and another cycle starts. In the terminology of control

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theory, the amount of cash behaves like a one trigger-one target impulse control, where the money stock between controls is described by a Brownian motion (BM) with drift.

The second category of models concerns cash management by firms, pioneered in the 1966 paper of Miller and Orr (Miller and Orr, 1966). Firms differ from households in that they have daily cash inflow as well as daily expenditures, and because large financial transactions are more likely. The former characteristic implies that the control policy is of the two triggers-two targets type, so that the money stock is adjusted when hitting either a low or a high trigger value. The existence of instantaneous large transactions makes BM alone inappropriate as a description of the stochastic process of the money stock between controls.

In this paper we present a unified analysis of cash management. Changes of the money stock, between controls, are described as a superposition of a BM and a compound Poisson process (CPP) with positive and negative jumps. Whenever this money stock process hits or crosses one of the two trigger levels, it is impulse controlled back to one of two prespecified targets. This enables us to bridge the somewhat artificial distinction between "household" and "corporate" cash management and to obtain a more general demand model.

An important application that suits our model well is management of foreign exchange reserves by central banks. The standard approach for the study of this problem is the Baumol–Tobin model of money demand, where the dynamics of international reserves is described by a BM (Frenkel and Jovanovic, 1981; Ben-Bassat and Gottlieb, 1992; Flood and Marion, 2001). This is a good description in normal times when international transactions are small relative to the stock of reserves. However, a financial crisis might cause very large outflows of international reserves in a very short period. Mexico (1995), Korea (1998), Brazil (1998), and Argentina (2002) are examples of countries that lost a large fraction of their reserves (20–30 billion dollars, 50–90% of the reserves) during a few days of crisis. Such a critical loss can be well described by a negative jump of a compound Poisson, giving rise to a superposition of a BM and a CPP to model the dynamics of reserves.

The approach presented here can be used in other applications. Dixit (1991) surveys several economic applications of impulse control. Our model seems to be also suitable to study demand for capital, or physical investment, with the help of "real options" (Dixit and Pindyck, 1994; Bar-Ilan and Strange, 1996). Demand for labor with hiring and firing cost, as in Bentolila and Bertola (1990), is a similar application. As further examples that are studied extensively in the literature we mention pricing decisions of firms that face menu costs and stochastic inflation (Caplin and Leahy, 1991), control of currency exchange rates inside a band (Krugman, 1991) and demand for durable goods (Grossman and Laroque, 1990).

In all the studies quoted above, the underlying stochastic process governing the state variables in the various applications is either a BM or a jump process. However, in most cases a more realistic description would be a superposition of both processes, combining continuous infinitesimal movements with occasional jumps. Capital stock can be depreciated in a smooth fashion that matches a BM, but can also change abruptly due to sudden death of part of the capital; the money stock is affected by small as well as large deposits and withdrawals; the number of employees can change