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Stochastic models for broker inventory in dealership markets with a cash management interpretation

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Abstract

We study the problem of a broker in a dealership market whose buffer content (cash flow) is governed by stochastic price-dependent demand and supply. Three model variants are considered. In the first model, buyers and sellers (borrowers and depositors) arrive independently in accordance with price-dependent compound Poisson streams. The second and the third models are two variants of diffusion approximations. For a certain natural revenue function, taking into account the trade-off between holding and shortage costs (opportunity loss and interest rates), we define relevant cost functionals that lay the groundwork for optimization purposes. The approach in analyzing and computing the cost functionals is based on the optional sampling theorem applied to a certain martingale.

Keywords: Dealership (spot) market; Buffer; Cash flow; Stochastic demand and supply; Cost functionals; Martingales

1. Introduction

In the first interpretation of this paper, we study the behavior of a broker who purchases and sells some commodity such as oil, metal, etc., on the open spot market. In an initial model, we take the demand process, represented by arrivals of commodity buyers, to be a compound Poisson process with exponential jump sizes (amounts bought from the broker). There is also an independent compound Poisson process with exponential jump sizes (amounts offered to the broker formed by a stream of commodity sellers). The difference between the amounts offered and demanded yields a net inventory process for the broker which may be positive or negative. In the absence of control, the content level of the inventory fluctuates as a continuous-time random walk. However, for a positive inventory position, the broker incurs a "holding" cost associated with tied-up capital or borrowing, while for negative position, there are shortage costs related to the provision of alternative suppliers or other special arrangements. Large positive or negative levels of inventory imply high attendant holding or shortage costs. The broker can influence the size of the inventory level by exercising some control over the sale and purchase prices. In this way, increasing the sale price will tend to stifle demand, while an increased purchase price will induce additional offerings to the broker.

Surprisingly, as far as known to the authors, and despite the natural motivation of the problem, there appears to have been no attempt in the operations research (OR) literature to study variants or ramifications of this OR problem

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class. Perhaps this is so, because traditional OR inventory models assume random demand under specified ordering policies, while, indeed, in this problem, the supply is also a random process.

Problems of this type are also analyzed in the literature in the context of cash management modeling. Our problem can then be viewed as a cash management model in the following way. The content level is interpreted as the cash flow process, with borrowers and depositors arriving in accordance with two compound Poisson streams. The holding costs are the opportunity loss rate for cash held within the fund and the shortage cost is the amount of expenditure per period incurred by interest on the shortage.

Although it would be possible to model such a control mechanism with many possible values for deposits and withdrawals as a function of the cash level, the corresponding model structure and solution would be exceedingly complex. Accordingly, we will allow some degree of dependence to facilitate the discussion and say that a pricing, or interest rate structure is imposed that will give depositors one offering rate when the cash level is positive and another when it is negative. Similarly, the interest rate for borrowers will depend on whether or not the cash fund is positive or negative.

The OR stochastic inventory perspective is perhaps more general than that of the cash balance context because cash can also be interpreted as a particular commodity. Therefore, in the sequel we model our problem as a general controlled dealership market problem. Formally, we consider a broker maintaining the stochastic inventory level process $W = \{W(t), t \geq 0\}$ defined over the state space $(-\infty, \infty)$. Level depletions result from buyers arriving at rate η as a compound Poisson process, $\{M(t), t \geq 0\}$, with demand sizes exponential (ν). Level increases result from sellers arriving at rate λ as a compound Poisson process, $\{N(t), t \geq 0\}$, with exponential (μ) amounts offered. Clearly, $W(t) = N(t) - M(t)$ is a continuous time random walk.

Define the process drift by $\gamma = (\lambda/\mu) - (\eta/\nu)$. Since the selling price naturally exceeds the purchase price, the broker is willing to buy and sell as much as possible. Consequently, in the absence of price control, either $W(t) \to \infty$ as $t \to \infty$, if $\gamma > 0$ or $W(t) \to -\infty$ as $t \to \infty$ if $\gamma < 0$, w.p. 1. However, stability can be assured by setting prices according to the state of the system; in this way, the broker can regulate the inventory level process exploiting a particular functional relationship between prices and, say, the demand arrival rate λ. Without loss of generality, we will assume two values for λ: λ^+ whenever $W(t) > 0$, and λ^- whenever $W(t) < 0$, as reflecting the supplier behavior induced by setting prices according to the value of $W(t)$ (alternatively, the controller may switch between two values of η). The values λ^+ and λ^- satisfy the requirement that the drifts $\gamma^+ = (\lambda^+/\mu) - (\eta/\nu) < 0$ and $\gamma^- = (\lambda^-/\mu) - (\eta/\nu) > 0$. In this way, W will alternate between positive and negative values, so that $N(t)$ is now a modulated compound Poisson process with the demand rate switching between λ^+ and λ^- according to an alternating renewal process.

In Section 2, we develop Model 1, which is valid only for buyer and seller processes which are compound Poisson with exponential jump sizes. Furthermore, Model 1 requires drifts $\gamma^+ < 0$ and $\gamma^- > 0$ to ensure stability of the process. If either of these requirements is not maintained, then alternative approaches must be sought. Accordingly, to incorporate non-exponential jumps, we will introduce, and limit our discussion to, heavy traffic approximation models with various drift mixtures for γ^+ and γ^-. These diffusion models 2 and 3 will be explored in Sections 3 and 4, respectively. The steady-state analysis of each model is obtained as a by-product of Sections 2 and 3 and is presented in Section 5.

In fact, most of the relevant literature to this model class uses the motivation of stochastic cash management and applies diffusion approximations. Alistair and Robertson (1996) used a diffusion process to study the behavior of a firm facing liquidation if their internal cash balance falls below some threshold. They optimized a model under the conflicting claims between the desire to pay dividends and the need to retain cash as a barrier against possible liquidation. Browne (1995) considered a firm facing an uncontrollable stochastic cash flow, and studied the optimal investment decision by taking into account the trade-off between holding a risky stock or cash. Bardhan (1994) studied a problem of an investor, endowed with a deterministic level of initial capital, who must use the endowment to consume and invest in the financial market. An example would be an investment firm with a cash flow commitment that is subject to regulatory capital constraints. Heathcote and Huesler (1991) considered a problem of finding the distribution of the rate of return on an investment when the cash flow is described by a Gaussian process. Smith (1989) considered a stochastic, time varying interest rate in a continuous-time inventory