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Multi-objective evolutionary computation and fuzzy optimization

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Abstract

In fuzzy optimization it is desirable that all fuzzy solutions under consideration be attainable, so that the decision maker will be able to make "a posteriori" decisions according to current decision environments. No additional optimization runs will be needed when the decision environment changes or when the decision maker needs to evaluate several decisions to establish the most appropriate ones. In this sense, multi-objective optimization is similar to fuzzy optimization, since it is also desirable to capture the Pareto front composing the solution. The Pareto front in a multi-objective problem can be interpreted as the fuzzy solution for a fuzzy problem. Multi-objective evolutionary algorithms have been shown in the last few years to be powerful techniques in solving multi-objective optimization problems because they can search for multiple Pareto solutions in a single run of the algorithm. In this contribution, we first introduce a multi-objective approach for nonlinear constrained optimization problems with fuzzy costs and constraints, and then an "ad hoc" multi-objective evolutionary algorithm to solve the former problem. A case study of a fuzzy optimization problem arising in some import–export companies in the south of Spain is analyzed and the proposed solutions from the evolutionary algorithm considered here are given.

Keywords: Fuzzy optimization; Multi-objective optimization; Evolutionary computation

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1. Introduction

1.1. Fuzzy optimization

It is well known that optimization problems arise in a variety of situations. Particularly interesting are those concerning management problems as decision makers usually state their data in a vague way: "high benefits", "as low as possible", "important savings", etc. Because of this vagueness, managers prefer to have not just one solution but also a set of them, so that the most suitable solution can be applied according to the state of existing decision of the production process at a given time and without increasing delay. In these situations, fuzzy optimization is an ideal methodology, since it allows us to represent the underlying uncertainty of the optimization problem, while finding optimal solutions that reflect such uncertainty and then applying them to possible instances, once the uncertainty has been solved. This allows us to obtain a model of the behavior of the solutions based on the uncertainty of the optimization problem.

Fuzzy constrained optimization problems have been extensively studied since the seventies. In the linear case, the first approaches to solve the so-called fuzzy linear programming problem appeared in [21,24]. Since then, important contributions solving different linear models have been made and these models have been the subject of a substantial amount of work. In the nonlinear case [1,8,19] the situation is quite different, as there is a wide variety of specific and both practically and theoretically relevant nonlinear problems, with each having a different solution method.

Fuzzy optimization problems also appear in literature with multiple objectives [11], and, typically, fuzzy logic has been used by numerous authors to solve multi-objective optimization problems [16,20].

However, for a number of reasons (necessity for managers, practical applications, theoretical aspects, etc.), the consideration of fuzzy nonlinear programming problems makes perfect sense. We consider the following general fuzzy nonlinear programming problem:

$$\begin{aligned} &\max f(\mathbf{x}, \tilde{\mathbf{c}}) \\ &\text{s.t.:} \\ &\qquad g_j(\mathbf{x}, \mathbf{a}_j) \lesssim b_j, \quad j = 1, \ldots, m, \end{aligned} \tag{1}$$

where $\mathbf{x} = (x_1, \ldots, x_n)$ is a real-valued parameter vector, with $x_i \in [l_i, u_i] \subset \mathbb{R}$, $l_i \geqslant 0$, $i = 1, \ldots, n$, $f(\mathbf{x}, \tilde{\mathbf{c}})$ is an arbitrary function depending on a fuzzy cost vector $\tilde{\mathbf{c}} = (\tilde{c}_1, \ldots, \tilde{c}_p)$, $g_j(\mathbf{x}, \mathbf{a}_j)$ are arbitrary functions depending on a coefficient vector $\mathbf{a}_j = \left(a_j^1, \ldots, a_j^q\right)$, $a_j^s \in \mathbb{R}$, $s = 1, \ldots, q$, and $b_j \in \mathbb{R}$. We assume fuzzy costs are characterized by membership functions of the following form:

$$\mu_k(v) = \begin{cases} 0 & \text{if } v \leqslant r_k \text{ or } v \geqslant R_k, \\ h_{1k}\left(\dfrac{v - r_k}{\underline{c}_k - r_k}\right) & \text{if } r_k \leqslant v \leqslant \underline{c}_k, \\ h_{2k}\left(\dfrac{R_k - v}{R_k - \bar{c}_k}\right) & \text{if } \bar{c}_k \leqslant v \leqslant R_k, \\ 1 & \text{if } \underline{c}_k \leqslant v \leqslant \bar{c}_k, \end{cases} \tag{2}$$