Swarm intelligence based techniques for digital filter design

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\section*{Introduction}

Studies by Fang et al. \cite{Fang2013} conforms the findings by the researchers of \cite{Bijay2013-1, Bijay2013-2, Bijay2013-4, Bijay2013-5, Bijay2013-10, Bijay2013-13}. This work can be treated as extension of the works done in \cite{Fang2013}. For ease of reading, we have reproduced the work as first part of this article that paves a way for the next part of the article.

The problem, design of IIR filters, error surface in most of the cases is non-quadratic and multi-modal \cite{Fang2013}. In a multi-modal error surface, global minima can be achieved by a global optimization technique. Bio-inspired algorithm, Genetic Algorithm (GA) \cite{Fang2013}, and its hybrids \cite{Bijay2013-5, Bijay2013-10, Bijay2013-13} have been used for digital IIR filter design. Because of disadvantages of GA (i.e., lack of good local search ability and premature convergence) invited Simulated Annealing (SA) \cite{Bijay2013-4} for use in the digital IIR filter design. Again, standard SA is very slow and also requires repeated evaluations of cost function to converge to the global minima. Hence, Differential Evolution (DE) \cite{Bijay2013-11} finds a scope in digital IIR filter design. But, the problem with DE algorithm is that it is sensitive to the choice of its control parameters.

Particle Swarm Optimization (PSO) \cite{Bijay2013-12, Bijay2013-13} also used in the problem of digital IIR filter design. PSO can be easily implemented and also outperforms GA in many of the applications. But, as the particles in PSO only search in a finite sampling space, PSO may easily be trapped into local optima. One useful modification of PSO is Quantum-Behaved Particle Swarm Optimization (QPSO) \cite{Bijay2013-14, Bijay2013-15, Bijay2013-16, Bijay2013-17, Bijay2013-18}. QPSO uses the merits of quantum mechanics and PSO.

The problem of digital IIR filter design is a minimization problem that can be solved by PSO and QPSO. In addition, the parameters of filter are real coded as a particle and then the swarm represents all the candidate solutions.

In \cite{Fang2013}, Fang et al. proposed one modification to the standard version of QPSO. This modification, QPSO-M, is proposed by introducing a random vector in QPSO in order to enhance the randomness and global search ability. In this paper we propose a modification to PSO, where, the constriction factor is allowed to change in each of the iterations with a linearly decreasing $k$. Importance and utility of proposed modifications validated by novel application to design of IIR filter. Three examples for the purpose of filter design are tested and compared with PSO and QPSO.

Rest part of the paper is organized as: problem of filter design is outlined in section ‘Problem statement’. PSO, QPSO and proposed modifications are discussed in section ‘Methodologies’ followed by simulation examples in section ‘Simulation results’. Finally, the paper is concluded in section ‘Conclusion’.

\section*{Problem statement}

In this paper, we have used the same system structure as that used in \cite{Fang2013} and reproduced in this section. Here, the model of IIR filter is considered to be same as that of the autoregressive moving average (ARMA) model. The model can be defined by the difference equation \cite{Fang2013}:

\[ y(k) + \sum_{i=1}^{M} b_i y(k-i) = \sum_{i=0}^{L} a_i x(k-i) \]  

\[ (1) \]
where $x(k)$ represents input to the filter and $y(k)$ represents output from the filter. $M (\geq L)$ is order of the filter, $a_i$ and $b_i$ are the adjustable coefficients. The transfer function of this model takes following general form:

$$H(z) = \frac{A(z)}{1 + B(z)} = \frac{\sum_{i=0}^{L} a_i z^{-i}}{1 + \sum_{i=1}^{M} b_i z^{-i}}$$  \hspace{1cm} (2)

The problem of IIR filter design can be considered as an optimization problem, where, the mean square error (MSE) is taken as the cost function. Hence, the cost function is:

$$J(\omega) = E[e^2(k)] = E[d(k) - y(k)]^2$$  \hspace{1cm} (3)

Here, $d(k)$ is the desired response of the filter, $e(k) = d(k) - y(k)$ is the error signal. Using two sets of coefficients, $[a_i]_{i=0}^{L}$ and $[b_i]_{i=1}^{M}$ we can get the composite weight vector for the filter. This is defined as:

$$\omega = [a_0, \ldots, a_L, b_1, \ldots, b_M]^T$$  \hspace{1cm} (4)

The objective however is to minimize MSE (3). This is achieved through adjustment of $\omega$. Because of difficulty in realizing the ensemble operation, the cost function (3) can be replaced by the time-averaged cost function:

$$J(\omega) = \frac{1}{N} \sum_{k=1}^{N} e^2(k)$$  \hspace{1cm} (5)

Here, $N$ represents the number of samples used.

**Methodologies**

This section discusses on optimization algorithms used in this paper for the problem of digital IIR filter design.

**QPSO**

QPSO [14,15] is a global search technique and based on social behavior of animals. In QPSO, each particle is a potential solution to a problem. In M-dimensional space, particle position is defined as $X_i = (x_{i1}, \ldots, x_{id}, \ldots, x_{iM})$ and velocity is defined as $V_i = (v_{i1}, \ldots, v_{id}, \ldots, v_{iM})$. Particles can remember their own previous best position. The velocity and position updation rule for particle $i$ at $(k+1)$th iteration uses the relations:

$$v_{ik}^{k+1} = w \cdot v_{ik} + c_1 \cdot r_{1i} \cdot (p_{g} - x_{ik}) + c_2 \cdot r_{2i} \cdot (p_{k} - x_{ik}); \quad r_{1i}, r_{2i} \in U(0, 1)$$

$$x_{ik}^{k+1} = x_{ik} + v_{ik}^{k+1}$$  \hspace{1cm} (6)

Here, $c_1$ (cognitive) and $c_2$ (social) are two positive constants that controls respectively the relative proportion of cognition and social interaction. Vector $P_g = (p_{g1}, \ldots, p_{gd}, \ldots, p_{gM})$ is the previous best position (the position giving the best fitness value) of particle $i$, which is called $p_{best}$. And vector $P_k = (p_{k1}, \ldots, p_{kd}, \ldots, p_{kM})$ is the best position discovered by the entire population, which is called $g_{best}$. Parameter $w$ is the inertia weight and the optimal strategy to control it is to initially set to 0.9 and reduce it linearly to 0.4 [14]. QPSO is derived from fundamental theory of particle swarm having the merits of quantum mechanics. In the QPSO algorithm with $M$ particles in $D$-dimensional space, the position of particle $i$ at $(k+1)$th iteration is updated by:

$$x_{ik}^{k+1} = p_{gi}^{k} + \alpha \cdot |\mathcal{M}_{ij}^{k} - x_{ik}^{k}| \cdot \ln \left( \frac{1}{u_{i}^{k}} \right) \cdot u_{i}^{k} \in U(0, 1)$$  \hspace{1cm} (7)

$$p_{gi}^{k} = \varphi_{gi}^{k} + p_{i}^{k} + (1 - \varphi_{gi}^{k}) \cdot p_{g}^{k}; \quad \varphi_{gi}^{k} \in U(0, 1)$$  \hspace{1cm} (8)

$$\mathcal{M}_{ij}^{k} = \left( \frac{M}{M} \sum_{i=1}^{M} p_{1i}^{k} \ldots \ldots \sum_{i=1}^{M} p_{M}^{k} \right)$$  \hspace{1cm} (9)

Here, parameter $\alpha$ is called contraction–expansion (CE) coefficient. $P_g$ and $P_k$ have the same meanings as those in PSO. $MP$ is called Mean Best Position, which is defined as the mean of the $p_{best}$ positions of all particles.

Search space of QPSO is wider than that of PSO because of introduction of exponential distribution of positions. Also, unlike PSO, where each particle converges to the global best position independently, improvement has been introduced by inserting Mean Best Position in QPSO. Here, each particle cannot converge to the global best position without considering its colleagues because that the distance between the current position and $MP$ determines the position distribution of the particle for the next iteration.

**Modified QPSO**

This section reproduces the modification proposed in [1]. Through QPSO finds better global search ability than PSO, still may fall into premature convergence [16] similar to other evolutionary algorithms in multi-modal optimization like GA and PSO. This results in great loss of performance and yield sub-optimal solutions. In QPSO, in a larger search space, diversity loss of the whole population is also inevitable due to the collectiveness although. From Eq. (2), it is evident that when $|M_{p_{1}^{k}} - x_{ij}^{k}|$ is very small, the search space will be small and $X_{ij}$ cannot go to a new position in the subsequent iterations. The explorative power of particles is also lost and the evolution process may become stagnant. This case even may occur at an early stage when $|M_{p_{1}^{k}} - x_{ij}^{k}|$ is zero. Also, in the latter stage, the loss of diversity for $|M_{p_{1}^{k}} - x_{ij}^{k}|$ is a problem to be considered. In order to prevent this undesirable difficulty, we in this paper construct a random vector. This random vector will replace $|M_{p_{1}^{k}} - x_{ij}^{k}|$ with a certain probability $CR$, according to the difference between two positional coordinates that are re-randomized in the problem space, and the value of the new position updating equation becomes:

$$\tilde{\delta} = m_{u}x_{ik} - m_{u}x_{ik}, \quad x_{ij}^{k+1} = p_{gi}^{k} + \alpha \cdot \tilde{\delta} \cdot \ln \left( \frac{1}{u_{i}^{k}} \right)$$  \hspace{1cm} (10)

where $m_{u}x_{ik}$ and $m_{u}x_{ik}$ are two random particles generated in the problem space. Because of introduction of the random vector, the particles may leave from the current position and may be located in a new search domain.

The modified version of QPSO is termed here as QPSO-M. The procedure, used in this paper is outlined as:

**Step 1:** Initialize particles with random position and set the control parameter $CR$.  
**Step 2:** For $k = 1$ to maximum iteration, execute the following steps.  
**Step 3:** Calculate the mean best position $M_{P}$ among the particles.  
**Step 4:** For each particle, compute its fitness value $f(x_{ki})$. If $f(x_{ki}) < f(P_{ki})$, then $P_{ki} = x_{ki}$.  
**Step 5:** Select best position $P_{ki}$ among particles.  
**Step 6:** Generate a random number, in the range $[0, 1]$.  
**Step 7:** If $RN < CR$ then update the position according to (7)-(9), else according to (8) and (10).

**Proposed modification with time dependant constriction factor (PSO-LD)**

Recent works in [17,18] indicate that the use of a “constriction factor” may be necessary to insure convergence of the PSO. A
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