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An algebraic model for implementing expert systems based on the knowledge of different experts

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Abstract

The aim of this paper is to expound an original algebraic model for managing the knowledge provided by different expert humans when developing expert systems. This model is conceived as an extension of classical propositional logics in which each proposition is associated with a set of human experts who agree with it. In our model, the logical notions of tautological consequence and consistency of a set of formulae are reformulated taking into account the criteria and the knowledge of the different experts. The core of the paper is related to the discovery of a remarkable relation between these redefined logical concepts with the calculation of Groebner bases on an ideal of polynomials.

Keywords: Multivalued logics; Groebner bases; Symbolic computing

1. Introduction

Expert systems are computational programs on a certain domain which try to simulate the decisions that human experts on this domain would take. In the last years, much research has been focused on developing both techniques for representing the human knowledge in a computer and techniques for reasoning automatically.

An interesting way for representing knowledge in an expert system is based on describing it through propositional logic. By means of a mathematical result [7] based on previous work [1,5,9,10,17], the question about what this kind of expert systems based on propositional logic can be transformed to an algebraic problem [8,18]. In this way, expert systems based on propositional logic may be very easy implemented by means of a computer algebra system like CoCoA [15] or Polybori [2]. Making use of this result, different expert systems have been so far developed in recent years [12,13,16,14,19].

In some interesting domains very related to the development of experts systems, like medicine, different human experts may have different criteria or opinions in some points which may lead to different conclusions or which may be even contradictory. In order to manage the different criteria of these experts, we have extended the propositional logic

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redefining the concepts of consistence and tautological consequence. Taking into account this logical concepts, we have obtained a relation to the calculation of Groebner basis. This results allows us to deal algebraically the development of this expert systems which takes into account the criteria of different human experts.

In Section 2, we will give an overview of the expert systems based on Boolean propositional logic and the algebraic approaches obtained for implementing these expert systems in a computer algebra system. In Section 3 we will redefine the concepts of tautological consequence and consistence in order to manage different knowledge source. In Section 4, we will show a new algebraic result which allows us to implement expert systems based on the criteria of different human experts by means of computer algebra systems. In Section 5, we show an example clarifying the concepts and results obtained in this paper. In Section 6, we describe the possible applications of our model. Finally, in Section 7, we will summarize our conclusions.

2. Introductory notes about expert systems and computer algebra

In this section, we will introduce basic concepts related to Expert Sytstems based on logic propositions and computer algebra. As we will see, the concept of Groebner basis (see Section 2.2) is highly important to establish a relation between these two fields.

2.1. Notes about expert systems based on boolean propositional logic

In this section, we will describe some outlines of a Expert Systems whose knowledge is represented by formulae in Boolean propositional logic.

In Boolean propositional logic, given a finite set of atomic propositions $X_1, \ldots < *** >, X_m$, we may build formulae (Definition 1) using the Boolean connectives $\neg$, $\vee$, $\wedge$, and $\rightarrow$ (although the connective $\rightarrow$ may be derived from the other connectives, we have included it due to its importance in expert systems).

Definition 1 *(Formula).* Let $X_1, \ldots, X_m$ be variables. A formula is defined recursively as follows:

- $\top$ (representing 'true')
- $\bot$ (representing 'false')
- X_i, where $X_i \in X_1, \ldots, X_m$ is a variable (also it is usually called as proposition)
- $\neg B$, where B is a formula
- $B \wedge C$, where B and C are formulae
- $B \vee C$, where B and C are formulae
- $B \rightarrow C$, where B and C are formulae

We will make use of $\mathcal{C}$ to denote the set of formulae. By its importance, we distinguish formulae with the form $(A_1 \wedge A_2 \wedge \ldots \wedge A_k) \longrightarrow (A_{k+1} \vee \ldots \vee A_n)$ where $n \geq 2$, and each formula $A_1, \ldots A_n$ is either an atomic proposition (X) or the negation of an atomic proposition ($\neg X$). This kind of formulae are termed as *rules*. As may be seen, any formula with at least a connective $\vee$, $\wedge$ or $\rightarrow$ in Boolean propositional logic may be described (rewritten) as a rule. On this basis, we may consider that a expert system based on Boolean propositional logic is composed of these three elements:

Input The input of a expert system is concerned with the information related to the environment of the expert system. Since environment may change, this information is also subject to change as times goes by.

This information is described by means of a finite number of different atomic propositions: such propositions are termed as *input atomic propositions*.

Output The output of an expert system is concerned with the information inferred by the expert system which will be useful for performing actions in the environment (in this paper, each output would stand for a possible dangerous situation). It is described by means of a finite number of atomic propositions which we shall call *output atomic propositions*.

Knowledge-base The knowledge-base of the expert system is concerned with the information contained in the system, which is used along with the input of the expert system to infer the output of the system. In an