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Solution of fuzzy integrated production and marketing planning based on extension principle

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ABSTRACT

The integration of production and marketing planning is crucial in practice for increasing a firm's profit. However, the conventional inventory models determine the selling price and demand quantity for a retailer's maximal profit with exactly known parameters. When the demand quantity, unit cost, and production rate are represented as fuzzy numbers, the profit calculated from the model should be fuzzy as well. Unlike previous studies, this paper develops a solution method to find the fuzzy profit of the integrated production and marketing planning problem when the demand quantity, unit cost, and production rate are represented as fuzzy numbers. Based on Zadeh's extension principle, we transform the problem into a pair of two-level mathematical programming models to calculate the lower bound and upper bound of the fuzzy profit. According to the duality theorem of geometric programming technique, the two-level mathematical program is transformed into the one-level conventional geometric program to solve. At a specific α-level, we can derive the global optimum solutions for the lower and upper bounds of the fuzzy profit by applying well-developed theories of geometric programming. Examples are given to illustrate the whole idea proposed in this paper.

1. Introduction

In high-tech markets, Moore's Law operates unforgettably: Every 18 months or so, improvements in technology double product performance at no increase in price. Today the technological innovation is speeded up than ever. The selling price, purchase cost, and demand quantity of a technology product become more and more uncertain in the market. For example, the component costs and selling price of personal computer assembly industry are decreasing at a sustained and significant rate (Khouja, Park, & Saydam, 2005). The integration of production and marketing functions has been recognized to be crucial in practice for increasing a firm's profit and decreasing their conflicts by reducing losses incurred from separate decision makings.

Pricing and lot sizing are two important strategies that concern simultaneous determination of an item's price and lot size to maximize a firm's profit for constant but price-dependent demands over a planning horizon. For increasing a firm's profit and decreasing the conflicts by reducing losses incurred from separate decision makings of production and marketing, the integration of production and marketing functions has been recognized to be crucial in practice. Different from the classic production and marketing planning, the demand is typically determined as a decreasing power function of the selling price with constant elasticity as in monopolistic pricing situations (Lee, 1993). In such a situation, geometric programming technique is an efficient and effective method to solve nonlinear programming problem with the terms in power functional form in the objective function and constraints (Beightler & Philips, 1976; Duffin, Peterson, & Zener, 1967).

The geometric programming technique has been applied to solve production and inventory problems. Worrall and Hall (1982) employ geometric programming technique to solve an inventory model with multiple items subject to multiple constraints. Cheng (1991) proposes an economic order quantity (EOQ) model with demand-independent unit cost and derived the optimal solution by employing geometric programming technique. Lee (1993) utilizes geometric programming techniques to determine the selling price and order quantity for a retailer. Kim and Lee (1998) investigate the fixed and variable capacity problems of jointly determining an item's price and lot size for a profit-maximization firm facing price-dependent demand. Jung and Klein (2005) employ the geometric programming technique to analyze three EOQ based inventory models under total cost minimization. Sadjadi, Oroujee, and Aryanezhad (2005) present an integrated production, marketing and inventory model which determines the production lot size, marketing expenditure and product's selling price through geometric programming. Mandal, Roy, and Maiti (2006) formulate an EOQ model with and without truncation on the deterioration term. The problem is converted to the minimization

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of a signomial expression with a posynomial constrain and is solved by modified geometric programming and nonlinear programming methods. Leung (2007) proposes an economic production quantity (EPQ) model with a flexible and imperfect production process and utilizes geometric programming technique to establish more general results using the arithmetic–geometric mean inequality. Liu (2007) considers the Cobb-Douglas production function with input quantity discount and employs geometric programming technique to derive the objective value for the profit-maximization problem.

Those studies share one common characteristic that the parameters in the models are precisely known. However, in real world applications, the parameters in the model might be inexact and imprecise in nature. There are also situations that the data cannot be collected without errors. Fuzzy set theory has been widely used in production and inventory management researches (for example, Chen, 2011; Chuu, 2009; Karakas, Koyuncu, Erol, & Kokangul, 2010; Liang, 2008; Liu, 2005, 2008, 2011; Vasant, 2006). Roy and Malti (1997) formulate a fuzzy single-objective lot-sizing model and the solution is derived by applying geometric programming. Mandal, Roy, and Maiti (2005) utilize the geometric programming technique to solve the fuzzy multi-item inventory problem where the multiple objectives are considered. Islam and Roy (2006) propose a fuzzy production lot-sizing model with the consideration of flexibility and reliability in production processes. Panda, Kar, and Maiti (2008) utilize the fuzzy stochastic programming and geometric programming techniques to solve multi-item EOQ problems under fuzzy and fuzzy stochastic resource constraints. Sadjadi, Ghazanfari, and Yousefli (2010) propose a pricing and marketing model with fuzzy parameters. In their model, there are three elasticity, namely, selling price, marketing expenditure, and lot size, are assumed to be fuzzy numbers. The fuzzy logic controller is designed to derive the optimum or near optimum values for all decision variables.

When the parameters are fuzzy numbers, the objective values should be fuzzy as well. In this study, we presume that the scaling constant to demand, scaling constant to unit cost, and scaling constant to production rate are convex fuzzy numbers. Based on Zadeh's extension principle, we develop the solution method for the fuzzy integrated production and marketing planning problem. At a specific α-level, a pair of two-level mathematical programming models is formulated to calculate the lower bound and upper bound of the fuzzy profit of the model. According to the duality theorem of geometric programming technique, the two-level mathematical program is transformed into the one-level conventional geometric program. Solving the transformed pair of geometric programming problems produces the interval of the profit at the specified α-level. Since they are exact optimum solutions, no heuristics are required. The associated selling price, marketing expenditure, and production lot size are then determined.

The rest of this paper is organized as follow. We first describe the problem and model formulation with fuzzy parameters. Next, a pair of two-level mathematical programs, which is based on extension principle, for calculating the bounds of the profit is formulated; we transform the two-level mathematical programs into one-level conventional geometric programs to solve. Then we use two examples to explain the idea of this paper. Finally, some conclusions are drawn from the discussion.

2. The problem

Consider a product is being introduced to customers. The selling price and marketing expenditure of the product have an impact on demand quantity. We suppose that the demand quantity d can be regarded as the function of price p and marketing expenditure m with the associated elasticity; that is, $d = Kp^{-\beta}m^{\gamma}$, where K is the scaling constant and β and γ are the price elasticity and expenditure elasticity, respectively. We assume $\beta > 1$, i.e., the price is elastic, and $0 < \gamma < 1$. The unit cost of this product is changing within a range and the supplier offers quantity discount. We represent the unit cost c as the decreasing function $c = Rq^{-\delta}$, $0 < \delta < 1$, in which q is the production lot-size and R is the scaling constant for unit cost. The discount factor δ is a very small value. If $\delta = 0$, it means no quantity discount. Let A and I be the setup cost and inventory carrying cost rate, respectively, and let μ be the production rate, which is assumed to increase as demand increases. Moreover, we consider that when the demand slows down, the production rate needs to decrease for maintaining a lower holding cost. In this study, we let $\mu = Vd$ with $V > 1$, that is, μ varies with d proportionally.

According to Sadjadi et al. (2005), we have the following formulation for the profit per unit time:

Profit = F = total revenue – marketing cost – production cost – set up cost – inventory holding cost

$$= pd - md - cd - Ad/q - Ic\left(1 - \frac{d}{\mu}\right)\frac{q}{2}, \tag{1}$$

$$= pd - md - cd - Ad/q - Ic\left(1 - \frac{d}{Vd}\right)\frac{q}{2}, \tag{2}$$

Let $S = 1 - 1/V$. Then Eq. (2) becomes:

$$F = pd - md - cd - Ad/q - IcS\frac{q}{2}, \tag{3}$$

$$= Kp^{-\beta+1}m^{\gamma} - Kp^{-\beta}m^{\gamma+1} - KRp^{-\beta}q^{-\delta}m^{\gamma} - AKp^{-\beta}q^{-1}m^{\gamma} - 0.5IRSq^{-\delta+1}, \tag{4}$$

In this model we have three assumptions, namely, instantaneous replenishment, no shortage cost, and batch production quantity. The objective of this model is to maximize the profit with decision variables p, q, and m.

In (4) all the coefficients must be precise. However, the scaling constants of K, R, and V in the functions of price, unit cost, and production rate, respectively, may not be measured precisely due to the fierce competitions and changing markets. Intuitively, if any of the scaling constants is imprecise and can be represented as a fuzzy number, the objective value should be a fuzzy number as well. Let $\widetilde{K}$, $\widetilde{R}$, $\widetilde{V}$, and $\widetilde{S} = [1 - 1/\widetilde{V}]$ denote the fuzzy counterparts of K, R, V, and S, respectively. Then, (4) becomes the following formulation with fuzzy parameters:

$$\widetilde{F} = \widetilde{K}p^{-\beta+1}m^{\gamma} - \widetilde{K}p^{-\beta}m^{\gamma+1} - \widetilde{K}\widetilde{R}p^{-\beta}q^{-\delta}m^{\gamma} - A\widetilde{K}p^{-\beta}q^{-1}m^{\gamma} - 0.5I\widetilde{R}\widetilde{S}q^{-\delta+1}, \tag{5}$$

Let $\mu_{\widetilde{K}}$, $\mu_{\widetilde{R}}$, and $\mu_{\widetilde{S}}$ be the membership functions of $\widetilde{K}$, $\widetilde{R}$, and $\widetilde{S}$, respectively. We have

$$\widetilde{K} = \{(k, \mu_{\widetilde{K}}(k)) | k \in \mathbf{K}\}, \tag{6}$$

$$\widetilde{R} = \{(r, \mu_{\widetilde{R}}(r)) | r \in \mathbf{R}\}, \tag{7}$$

$$\widetilde{S} = \{(s, \mu_{\widetilde{S}}(s)) | s \in \mathbf{S}\}, \tag{8}$$

where $\mathbf{K}$, $\mathbf{R}$, and $\mathbf{S}$ are the crisp universal sets of $\widetilde{K}$, $\widetilde{R}$, and $\widetilde{S}$. Denote the α-cuts of $\widetilde{K}$, $\widetilde{R}$, and $\widetilde{S}$ as:

$$K_{\alpha} = \left[K_{\alpha}^{L}, K_{\alpha}^{U}\right] = [\min_{k}\{(k, \mu_{\widetilde{K}}(k)) | \mu_{\widetilde{K}}(k) \geqslant \alpha\}, \max_{k}\{(k, \mu_{\widetilde{K}}(k)) | \mu_{\widetilde{K}}(k) \geqslant \alpha\}], \tag{9}$$

$$R_{\alpha} = \left[R_{\alpha}^{L}, R_{\alpha}^{U}\right] = [\min_{r}\{(r, \mu_{\widetilde{R}}(r)) | \mu_{\widetilde{R}}(r) \geqslant \alpha\}, \max_{r}\{(r, \mu_{\widetilde{R}}(r)) | \mu_{\widetilde{R}}(r) \geqslant \alpha\}], \tag{10}$$

$$S_{\alpha} = \left[S_{\alpha}^{L}, S_{\alpha}^{U}\right] = [\min_{s}\{(s, \mu_{\widetilde{S}}(s)) | \mu_{\widetilde{S}}(s) \geqslant \alpha\}, \max_{s}\{(s, \mu_{\widetilde{S}}(s)) | \mu_{\widetilde{S}}(s) \geqslant \alpha\}]. \tag{11}$$