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Strategic complementarities and search market equilibrium

Michael T. Rauh

Kelley School of Business, Department of Business Economics, Indiana University, 1309 East Tenth Street, Bloomington, IN 47405-1701, USA

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Abstract

In this paper, we apply supermodular game theory to the equilibrium search literature with sequential search. We identify necessary and sufficient conditions for the pricing game to exhibit strategic complementarities and prove existence of equilibrium. We then show that price dispersion is inherently incompatible with strategic complementarities in the sense that the Diamond Paradox obtains when firms are identical and is robust within the class of search cost densities that are small near zero and support strategic complementarities. We also show that a major criticism of the literature, that agents act as if they know the distribution of prices, can be justified in the sense of convergent best response dynamics.

JEL classification: D43; D83

Keywords: Diamond Paradox; Price dispersion; Search; Strategic complementarities

1. Introduction

In this paper, we apply concepts and tools from supermodular game theory to address three fundamental and long-standing open questions in the equilibrium search literature. The first question is: what is the nature and source of observed price dispersion, which has been extensively documented in real-world markets? At present, the search literature contains several distinct classes of models that can explain this phenomenon, including sequential search models such as Reinganum (1979), Rob (1985), and Stahl (1989, 1996); nonsequential search models such as Burdett and Judd (1983) and Janssen and Moraga-González (2004); and information clearinghouse models such as Baye and Morgan (2001). For a survey of this literature, see Baye et al. (2006).

In this paper, we consider the class of equilibrium search models with sequential search developed by Carlson and McAfee (1983) and Bénabou (1993), which allows for potential bilateral heterogeneity in consumers' search costs as well as firms' production costs. According to conventional wisdom, distilled from Rob's (1985) seminal contribution, dispersion can be a purely informational phenomenon in this class of models in the sense that it can be generated solely by heterogeneous search costs, without any heterogeneity in firms' production costs. An important aspect of his model, however, is that demand is assumed to be perfectly inelastic at one unit with infinite willingness to pay, which cannot be literally true with finite income. This assumption rules out *a priori* the Diamond (1971) equilibrium

E-mail address: mtrauh@indiana.edu.

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where identical firms almost all charge the common monopoly price, because the latter is not well-defined. As a result, dispersion occurs in Rob's model under quite general conditions.

In contrast, we show that when demand is potentially downward-sloping with finite willingness to pay, a Rob-type condition applies (the value of the density of search costs at zero is sufficiently small), and the pricing game exhibits strategic complementarities, the unique equilibrium with identical firms (same production cost) is the Diamond one. In that case, no amount of heterogeneity in search costs can overcome the Diamond Paradox as long as the distribution of search costs remains within the class that supports strategic complementarities. Under these conditions, it seems that price dispersion either requires an atom in the distribution of search costs at zero as in Stahl (1989, 1996) or some additional heterogeneity like heterogeneous production costs as in Reinganum (1979) or heterogeneous sampling probabilities as in Hortaçsu and Syverson (2004). Indeed, we show that a simple Reinganum-type condition on firms' costs ensures that all equilibria are dispersed. In that case, however, dispersion is not a purely informational phenomenon because it is driven at least in part by *technological* conditions.

Since its inception in the early 1960s, a major recurring criticism of the literature has been that consumers and firms act "as if" they know the distribution of prices. E.g.,

> These results depend on the assumption that the searcher behaves as if he knows the distribution of prices. In any economic context, this is a very bad assumption. Little is known about the nature of price distributions, and it seems absurd to suppose that consumers know them with any degree of accuracy.
>
> Rothschild (1974)

> Perhaps the most restrictive and least palatable assumption of the elementary search model is the supposition that the offer distribution F is known.
>
> Lippman and McCall (1981).

Although this criticism is valid for any static game-theoretic equilibrium concept, it applies with particular force to this literature, whose explicit aim after all is to characterize markets with informational imperfections. Moreover, it seems clear that real-world consumers (and firms) do not have anything close to the required information.

To address this issue, Bénabou and Gertner (1993) and Dana (1994) construct search models where firms have private information about their costs and consumers learn about the distribution of prices while they search, given their knowledge of the distribution of production costs as well as firms' pricing strategies. Although this approach generates important insights (e.g., about the link between dispersion and unanticipated inflation), these informational requirements are clearly very strong. Another approach is that of Rauh (1997), where agents only know finitely many moments of the distribution of prices. Although this is weaker than knowing the entire distribution (all infinitely many moments), it is still not clear how agents obtain this information, which is contemporaneous.

In this paper, we pursue a dynamic approach to this issue and show that "as if" knowledge of last period's distribution of prices can be sufficient in the sense of convergent best response dynamics, where consumers' reservation levels and firms' profit-maximizing prices are best responses against the distribution of prices in the previous period. In contrast, Hopkins and Seymour (2002) show that the mixed-strategy equilibria in Varian (1980) and Burdett and Judd (1983) are generally dynamically unstable with respect to a broad class of learning rules that includes fictitious play, which is analogous to best response dynamics. Although the aforementioned models can generate dispersion with little or no *ex ante* heterogeneity, their mixed-strategy equilibria tend to be unstable, whereas the model in this paper can possess stable dispersed equilibria, but in general dispersion requires heterogeneity in both search and production costs.

The final issue concerns existence of search market equilibrium. In this literature, the pricing game consists of a continuum of firms whose profits depend on their own prices as well as the *distribution* of prices in the market. Unfortunately, little seems to be known about games with this structure, so existence is a nontrivial problem and the main methodological innovation of the present paper is its appeal to supermodular game theory, as developed by Milgrom and Roberts (1990), Topkis (1979, 1998), and Vives (1990, 1999). Indeed, strategic complementarities seem quite natural in the current context involving price competition with a homogeneous good: if firm i maintains its price while all of its competitors raise theirs, then i should receive more visits from potential customers, allowing it to raise price as well.