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Research on spot power market equilibrium model considering the electric power network characteristics

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Abstract

Equilibrium is the optimum operational condition for the power market by economics rule. A realistic spot power market cannot achieve the equilibrium condition due to network losses and congestions. The impact of the network losses and congestion on spot power market is analyzed in this paper in order to establish a new equilibrium model considering the network loss and transmission constraints. The OPF problem formulated according to the new equilibrium model is solved by means of the equal price principle. A case study on the IEEE-30-bus system is provided in order to prove the effectiveness of the proposed approach.

Keywords: Spot power market; Equilibrium; Network losses; Congestion

1. Introduction

With the ongoing restructure and deregulation of power industry around the world, electric power industry has entered into an increasingly competitive era. It has became clear that it is possible to improve both economy and reliability of power system by market force. The basic theories for the power market, which must combine the operational rule in the power system with the market principles in economics, have aroused interest of many researchers.

Many models had been proposed to design and simulate a power market. They can be classified into two groups: (1) The auction design based on microeconomic theory. The primary idea is to conduct auction to match supply and demand [1–4] including the equal price method [5,6]. (2) Introducing the traditional economic dispatching arithmetic into the power market transactions [7,8] in order to determine optimal unit commitment and marginal price.

The simple and transparent auction process is the advantage of the first model but it is difficult to handle constraints, which are necessary to ensure that a power system is operated in security. The models in the second group are able to solve the optimization problem considering various constraints, but it is more complicated and lacks transparence. Therefore, the key issue is to integrate the market principle in economics into practical operation in the power system. Marcelino et al. [9] has tried to find the equilibrium point for the power pool auction by using Lagrangian relaxation approach.

The losses and congestion are two main characteristics of the electric power network related to the spot power transactions, therefore, the responsibilities for ISO (independent system operator) and PX (power exchange) are to make the power transactions based on the security operation of the power system. A large number of literatures in network losses allocation and congestion dispatching [10–13] are published up to now, which similarly overlook the application of the market principles.

The equilibrium model for the spot power market is described to indicate that equilibrium is the optimum con-

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dition for the power market, then the impact is analyzed and the equilibrium model designed for the spot power market is established considering the network losses and congestion. Therefore, the final OPF (optimal power flow) problem is solved by using equal price method in this paper. The following assumptions are proposed: (1) the ancillary services pricing problem is ignored when the active power transactions are made; (2) the supply offers are an increasing curve with the decreasing curve for the demand bids under power pool mode; (3) the bidding curves offered by market participants is uninterrupted and differentiable, that is reasonable because final arithmetic not related with the differentiability about the bidding curves whereas they are not differentiable at some points; (4) the unit commitment considered as obligation of the generation company is not included here.

The paper is organized as follows. Section 2 introduces the equilibrium model for the power market and its properties. The impact of the network losses to the power market equilibrium is analyzed and the new equilibrium model is established in Section 3. Section 4 introduces the power market equilibrium model under network congestion. The final power market equilibrium mathematic model is expressed as an OPF problem solved by equal price method in Section 5. Section 6 is a case study made at IEEE-30 nodes system.

2. The properties of market equilibrium

In a general commodity market, equilibrium means the optimum condition at which supply and demand achieve balance with maximum social benefits and equitable benefits allocation. The Walras equilibrium for a single commodity market can be described as follows:

$$F(C) = 0 \tag{1}$$

$$CF(C) = 0. \tag{2}$$

In (1) and (2), F is the net commodity demand representing the difference between supply and demand. C is the price of the commodity.

The equilibrium of supply and demand may also be realized in the power market showed in Fig. 1, where Curve 1 denotes the generation company's offers; Curve 2 denotes the consumer's biddings; C_{0M} is the equilibrium price; P_{0M} is the equilibrium power quantity. Thus, 0 is the equilibrium point. At this point, the market benefits achieved maximum and the market benefit allocation is equity. The shadow area above equilibrium price denotes the generation benefits while the area below denotes the consumer's benefits. It is important to simulate the power market because the power market cannot automatically achieve the equilibrium condition, which distinguishes a power market from a general commodity market.

In Fig. 1, it is assumed that there are a total of N participants with $1, 2, \ldots, M$ representing the number of suppliers and $M+1, M+2, \ldots, N$ the number of demands. The following optimal mathematic models can be used to express the equilibrium:

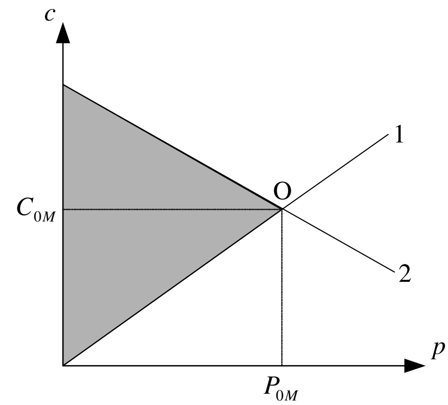

Fig. 1. Supply and demand equilibrium figure in the power market.

$$\min \quad z = \sum_{k=1}^{N} c_k p_k \tag{3}$$

$$\sum\nolimits_{k=1}^{N} p_k = 0 \tag{4}$$

$$\sum\nolimits_{k=1}^{N} c_k p_k = 0 \tag{5}$$

where p_k is the output power of k-th participant with price c_k, which is negative for consumer. It is easy to prove that $c_1 = c_2 = \cdots = c_N = C_{0M}$ and $z = 0$, then Eqs. (4) and (5) are uniform with formulas (1) and (2).

According to the discussion above, the properties of the power market equilibrium are summarized as follows: (1) the uniform marginal price is performed; (2) the balance of supply and demand is achieved and (3) the balance of income and expenses is achieved.

3. The impact of network losses on power market equilibrium

Eq. (6), instead of formula (4), represents the accurate expression of the active power balance with the network losses:

$$\sum_{k=1}^{N} p_k - P_{\mathrm{L}} = 0 \tag{6}$$

where P_{L} is the network losses in (6). The previous equilibrium no longer holds because of the presence network losses P_{L}, which must be fairly allocated among participants. The network losses pricing based on marginal cost theory is improper, because it leads to an imbalance between the practical and allocated network losses, thus imbalance between incomes and expenses. Therefore, a pricing based on embedded cost is necessary. The quadratic function below can be employed to calculate the network losses more accurately [14].

$$P_{\mathrm{L}} = \mathbf{P}^{\mathrm{T}}\mathbf{A}\mathbf{P} + \mathbf{B}\mathbf{P} + C \tag{7}$$

where $\mathbf{P}$ is the vector of injected power at nodes, $\mathbf{A}$, $\mathbf{B}$ are coefficient matrixes related with the state variables of the system while C is a constant, viz.:

$$P_{\mathrm{L}} = (\mathbf{P}^{\mathrm{T}}\mathbf{A} + \mathbf{B} + \mathbf{F}^{\mathrm{T}})\mathbf{P} \tag{8}$$