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Asset market equilibrium: A simulation

T. Fukiharu *

Faculty of Economics, Hiroshima University, Japan

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Abstract

This paper examines how an asset price is determined in a market, and how it changes as circumstances in the market change, making use of a standard asset price model. The motivation of the paper is to examine if the model can explain a *bubble economy* in which individuals are risk averse. It is known that if the *relative* risk aversion of an investor's utility function does not exceed 1 and is not decreasing, the equilibrium asset price declines when uncertainty increases with respect to the prospect of a dividend receipt. In this paper we examine if there is any utility function which provides the counter example, one in which the asset price rises despite increased uncertainty. Starting from a two period maximization problem with risk aversion, with *certain* dividends for the two periods, it is shown that if *uncertainty* is introduced for the *second* period, the *exponential* utility function provides this counter example. It is shown that in this particular case that the asset in question has the characteristics of a *Giffen* good *when the asset price is already high*. However, when uncertainty is introduced for *two* periods, the exponential utility function does not provide this counter example. Thus, when uncertainty is not as great, the income effect may raise the asset price despite increased uncertainty. It is also found that a *quadratic* utility function may explain the collapse of a bubble economy. The standard asset price model is formulated as the pure-exchange economy, without production. Finally, this paper points out that the existence of an asset price may be guaranteed even if production is incorporated into the standard model so long as a certain degree of uniformity is assumed about the distribution of the investors' initial possession of assets.

Keywords: Asset price; Bubble; Relative risk aversion; Giffen good; Simulation

1. Introduction

The main aim of this paper is to examine asset price variation when market conditions change. Historically, when a bubble emerged in an asset market, in many cases, there were opportunities to modify the circumstances which created it: e.g. in the Dutch tulip bubble. However, in a bubble economy, investors appear to be risk-takers. As for the stock bubble and the great crash 1929 in the USA (see refs. [1,6]). This paper focuses its attention on the explanation of a bubble economy when the standard assumption is made: i.e. investors are risk-averse. In a standard microeconomic exercise [7], it assumes the existence of one risky asset and one risk-less asset. When the investor's income increases, he or she reduces the *ratio* of investment in the risky asset, raising the one in the risk-less asset. This exercise is solved with the assumption of a quadratic utility function, a special type of risk-averse assumption. It appears that asset price bubbles cannot be explained in terms of this exercise if society were to consist only of the risk averse. The

* Tel.: +81 82 424 7295; fax: +81 82 424 7212.
E-mail address: fukito@hiroshima-u.ac.jp.

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standard exercise, however, makes no reference to the possibility of changes in *absolute* demand for the risky asset. Consequently, it does not explain how the asset price changes if market conditions change.

This paper examines how an asset price is determined in an asset market, and how it changes given changes in market conditions, utilizing the Lucas model [8], which incorporates *asset price* determination. The main motivation in ref. [8] was the *existence of* an *equilibrium* asset price. The motivation in this paper is to make use of *comparative statics* to examine if the asset price increases when the dividend from the asset becomes riskier, holding constant the expected value of the asset. I [3] concluded that the asset price would fall, given additional assumption relating to relative risk aversion on the part of investors. If these assumptions are not satisfied, however, what happens? This paper examines this scenario, utilizing a simulation approach.

2. Certain asset dividend case

The Lucas model constructs a household's asset purchasing plan over infinite periods in a pure exchange model, examining the existence of equilibrium in the asset market. Since the purpose of the present paper is to examine asset price variation under changed economic circumstances, we simplify the Lucas model as much as possible. In this section, we assume that there is an aggregate household, planning optimum asset possession and goods consumption over two periods, in a pure exchange model. There is one commodity and one asset. There is no production. Consequently, the model may be understood best if we suppose that the asset is a foreign asset, and the household's consumption from the dividend is procured from foreign trade. In this section one unit of the asset is assumed to yield the certain dividend, $\tilde{y}$. The assumption of certainty of dividend allows the investor to consume the same $\tilde{y}$ for the two periods. In the first period, the aggregate investor possesses the asset of the amount $z_0 = 1$. When p is the asset price, in terms of the consumption good, the aggregate investor possesses income of the amount $(\tilde{y} + p)z_0$ in the first period. The aggregate investor plans optimum consumption over the two periods: c_1 and c_2, and the purchase of the asset, z_1, in the belief that the purchase of the asset of the amount z_1 guarantees consumption in the second period, $c_2 = \tilde{y}z_1$, while the asset is bequeathed in the second period. Assuming $u(c)$ to be the utility function, the aggregate investor's behavior is expressed by the following maximization:

$$\max\ u(c_1) + \beta u(c_2) \quad \text{s.t. } c_1 + pz_1 \leq (\tilde{y} + p)z_0, \quad c_2 \leq \tilde{y}z_1 \tag{1.1}$$

where β is the discount factor.

From this maximization, we have demand functions, $c_1(p)$, $c_2(p)$, and $z_1(p)$. The *certain pure exchange* stock market equilibrium is defined by p^*, which satisfies

$$z_1(p^*) = 1, \quad c_1(p^*) = \tilde{y}, \quad \text{and} \quad c_2\,(p^*) = \tilde{y} \tag{2.1}$$

We have the following result, proved in ref. [3]. This result is no more than a fundamental theorem of *finance*; that the asset price is the present value of the stream of dividends.

Proposition 1. Suppose that

$$u'(c) > 0 \quad \text{and} \quad u''(c) < 0, \quad (c > 0). \tag{3.1}$$

Then, we have the following.

(i) $z_1(p) < 1$ holds when $\beta\tilde{y} < p$.
(ii) $z_1(p) > 1$ holds when $\beta\tilde{y} > p$.
(iii) Necessary and sufficient conditions for (2.1) to hold is $p^* = \beta\tilde{y}$.

From Proposition 1 it follows that $p^* = \beta\tilde{y}$ is the *stable* pure exchange stock market equilibrium. Assumption (3.1) is a standard one in microeconomics. In what follows, simulations are conducted by specifying a series of utility functions.