Electric Power Systems Research 80 (2010) 537–546

Contents lists available at ScienceDirect

Electric Power Systems Research

journal homepage: www.elsevier.com/locate/epsr

# Electricity market equilibrium of nonlinear power systems with reactive power control

A.G. Petoussis [a], X.-P. Zhang [b,*], S.G. Petoussis [c], K.R. Godfrey [a]

[a] School of Engineering, University of Warwick, Coventry, CV4 7AL, UK
[b] Department of Electronic, Electrical and Computer Engineering, University of Birmingham, Edgbaston, Birmingham, B15 2TT, UK
[c] Electricity Authority of Cyprus (EAC), Nicosia, CY-1399, Cyprus

ARTICLE INFO

*Article history:*
Received 19 December 2007
Received in revised form
10 September 2009
Accepted 24 October 2009
Available online 6 December 2009

*Keywords:*
AC network modeling
Electricity market equilibrium
Market power
Nodal prices
Reactive power control
Supply function equilibrium

ABSTRACT

The impact of reactive power control on the electricity market equilibrium is investigated. The effects of limitations on the reactive power generation and absorption, and load power factor adjustments, are examined using a novel electricity market equilibrium model that solves large-scale nonlinear power systems with asymmetric strategic firms. The algorithm implemented employs the linear supply function theory for bid-based pool markets. AC power flow analysis is used to represent the electricity network, incorporating variable price-responsive active and reactive load demands. The significance of the reactive power modeling in the electricity market equilibrium is demonstrated using the IEEE 14-bus and IEEE 118-bus systems. It is shown that variations on the reactive power in the system result in different market outcomes, as incentives are given to the strategic generating firms to alter their bidding strategies. The convergence characteristics of the IEEE 118-bus system are graphically presented and discussed to demonstrate the superior computational performance of the proposed algorithm in producing results under strict binding constraints and heavy transmission congestion conditions.


## 1. Introduction

Major reforms have taken place in the electricity sector around the world during the last two decades. The liberalization and deregulation of the power market has led to the replacement of its monopolistic structure by a more competitive environment. However, perfect competition was unlikely to be established due to the unique nature of the electricity market and the operational complexity of the generation, transmission and distribution of electrical energy. The result was the transformation of the electricity market into an oligopolistic structure with a small number of large participants, which have the ability of exercising market power to influence the overall market outcome for their benefit [1]. In order to provide insights regarding the oligopolistic electricity markets, power system designers were required to devise new market models based on the notion of economic equilibria [2] for the implementation of several market competition types. The technical aspects considered and the network representations employed vary between the models, depending on the scope of the particular research.

The model presented in this paper employs the linear supply function theory for the investigation of non-cooperative electricity market equilibria. The electricity network is represented using nonlinear AC power flow analysis in order to include the modeling of reactive power in the system and enhance the accuracy of the electricity market equilibrium solution. Such network modeling enables the examination of the electricity market equilibrium under different reactive power generation and absorption limits and different load power factors, showing the direct relation of the presence of reactive power in the system with the market equilibrium solution and the bidding strategies of the generating firms.

The structure of the rest of the paper is as follows. Section 2 provides an overview of the electricity market supply function equilibrium (SFE) models and the employment of different network representations, while it examines the reactive power interactions in the electricity market from the existing literature and outlines the key features of the proposed SFE model. The SFE market problem and the implementation of the solution procedure of this study are described in Section 3. Numerical results with discussions on the IEEE 14-bus and IEEE 118-bus systems for the investigation of the impact of the reactive power generation and absorption limits and of load power factor adjustments on the electricity market equilibrium are presented in Section 4. The performance of the proposed algorithm in solving large-scale nonlinear systems is examined in Section 5, where the convergence characteristics for

* Corresponding author. Tel.: +44 121 414 4298; fax: +44 121 414 4291.
*E-mail addresses:* a.petoussis@hotmail.com (A.G. Petoussis), x.p.zhang@ieee.org (X.-P. Zhang), SPetousis@eac.com.cy (S.G. Petoussis), k.r.godfrey@warwick.ac.uk (K.R. Godfrey).

0378-7796/$ – see front matter © 2009 Elsevier B.V. All rights reserved.
doi:10.1016/j.epsr.2009.10.022

the test cases performed are presented and discussed. Conclusions are drawn in Section 6.

## 2. Oligopolistic electricity market models

### 2.1. Supply function equilibrium

In 1989, Klemperer and Meyer [3] proposed a new oligopolistic model, supply function equilibrium (SFE), in the context of general economic theory. This method proposed that each player devises its strategy based on a supply function that relates both price and quantity. In order to analyze supply schedule equilibria in power markets with the participants competing in both quantities and prices, the application of the SFE concept in the electricity market was first proposed in [4,5]. To overcome the problems encountered in the electricity market modeling arising from the high computational complexities of the general SFE approach, the linear SFE model, for which linear marginal costs were implemented to handle asymmetric firms, was proposed in [6]. This model was further developed in [7–9], where it is proven that it can provide a more realistic depiction of the market equilibrium than the Cournot and Bertrand models.

Different approaches for the parameterization of the linear supply functions in bid-based pool markets were proposed. These include the variation of the intercept [10,11] or the gradient [10,12] of the linear supply function, or of both independently (arbitrarily) [13] or linearly dependent (variation of a bidding parameter that is multiplied with the linear supply function to form the bid) [14–16]. The approach for which the firms vary the intercept and gradient independently provides the ability to consider all the possible supply functions, but unique equilibrium exists only under strict operating conditions [11]. Therefore this application is limited due to implementation issues and a good alternative that illustrates successfully the market power interactions between the firms and the impact of the network constraints on the market outcome is the employment of the bidding parameter parameterization. Comparisons and discussions for the four parameterization techniques are provided in [13].

### 2.2. Network representations in SFE market models

Concerning the SFE models, the study in [4] did not consider any physical or operational aspects from the fundamental power system network theory, while [5,6,17,18] take into account the suppliers' maximum output capacity, without considering the transmission network. However, early investigations on the deregulated electricity markets have proven the relevance of network constraints with the potential of the firms to exercise market power, which results in different market outcomes. Case studies in [19] show that transmission costs isolate generators from competition with distant producers, while [20] discusses how the transmission capacity determines the degree in which the individual players compete with one another in deregulated electricity markets.

The first attempt to represent the power transmission network in a linear SFE market model was made in [10]. The linearized DC load flow analysis that considers Kirchhoff's voltage and current laws was employed and transmission line constraints along with the energy balance equations were included. Other studies [11,15] have used improved implementations of the DC model, including upper and lower active power generation capacity limits.

Although the majority of the literature related to SFE market models utilizes the linearized DC load flow model, the implementation of an AC model would have been more attractive in terms of accuracy and real-life market power modeling. Indeed, in [10] it is stated that there may be ways of exercising market power that are made possible only by the nonlinearities in AC transmission systems.

The study in [14] presents an AC power flow model in the linear SFE context, taking into account the active and reactive power flow equations, the MVA rating of the transmission lines and the bus voltage limits. Although reactive power was considered, its effects on the electricity market equilibrium received little attention. More advanced AC power flow models that consider, in addition to the above, active and reactive power generation capacities and transformer tap-ratio control are presented in [16,21] for the investigation of the linear SFE in electricity markets, in the presence of reactive power.

### 2.3. Reactive power interactions in the electricity market

As already mentioned, reactive power is seldom considered in the market equilibrium models. Discussions on how market power can arise from the presence of reactive power in the network have taken place in [22]. The discussion has raised the suspicion that since reactive power is of localized nature it may have a greater role in market power than that of active power.

The multi-leader–follower market model of [23] for which the leaders submit linear supply functions to a pool, shows that the strategic players can take advantage of the reactive power associated constraints, such as the MVA and voltage limits, to increase their profits. However, except that only one or two leaders were considered in the test cases, the reactive load demand in this model is not price-responsive but fixed to a predetermined value.

The effects of price-responsive variable active and reactive load demand on the linear SFE market model were addressed for the first time in [24]. The model provides the possibility to investigate how load power factor corrections and voltage control of the generator buses by means of reactive power variations in the system will affect the market equilibrium of a small 3-bus system. The results presented have shown that for different voltage control modes, the nodal prices and social welfare may increase or decrease depending on the presence of MVA transmission constraints in the network.

### 2.4. The proposed model

In the scheme employed in this paper, the linear SFE approach using the bidding parameter parameterization has been chosen to simulate the behavior of asymmetric strategic generating firms in large-scale nonlinear power systems, in the presence of reactive power in the network. The electricity network is represented using the AC model, taking into account the active and reactive power flows and network losses, the bus voltage limits, the generation active and reactive power capacity limits, the MVA transmission line constraints, the transformer tap-ratio control and the transformer MVA capacity limits.

The preliminary investigations described in the literature do not provide any insights on the effects on the electricity market related to the reactive power limits for either generation or absorption, or the impact of power factor variations in large power systems. The impact of the aforementioned reactive power control methods on the market outcome, under the consideration of price-responsive active and reactive load demand, is investigated. The major contributions of this study include the following:

(a) demonstration of the direct impact of the presence of reactive power in the system on the bidding strategies of the generating firms and the electricity market outcome, showing the significance of AC network representation in the equilibrium models;