Leverage management in a bull–bear switching market

Min Dai, Hefei Wang, Zhou Yang

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Abstract

Should an investor unwind his portfolio in the face of changing economic conditions? We study an investor’s optimal trading strategy with finite horizon and transaction costs in an economy that switches stochastically between two market conditions. We fully characterize the investor’s time dependent investment strategy in a “bull” market and a “bear” market. We show that when the market switches from the “bull” market to the “bear” market, complete deleveraging, reducing the degree of leverage, or keeping leverage unchanged may all be optimal strategies, subject to underlying market conditions. We further show that the investor may optimally keep leverage unchanged in the “bear” market, particularly so for illiquid asset. On the other hand, a lower borrowing cost in the “bear” market would prevent sell offs.

1. Introduction

Leverage, an instrument heavily used by financial institutions to amplify returns, is often being cited as the major cause of the recent financial crisis. If leverage is said to be part of the language of Wall Street, deleverage, however, has become the new “buzz” word since the financial crisis. Recent events highlight the importance of leverage management, and the painful decision to deleverage, especially when the market conditions may change drastically.1 Many institutional investors and university endowments managers debated heavily whether they should reduce leverage, or decrease their positions in risky asset holdings in a market downturn. A recent New York Times article reported University of Chicago endowment investment decisions, which caused a divergence of opinion among leading financial experts.2 While some managers keep their investments unchanged, despite the nosedive in the stock market, others prefer to “unload” their risky assets. Should an investor unwind his portfolio in the face of changing economic conditions?

In this paper, we address this deleveraging decision by studying an investor’s optimal investment problem with a finite horizon in a regime-switching setting. Specifically, we consider two market environments—a “bull” market, which is commonly associated with high asset returns and low volatility, or what we refer to as a high “variance-adjusted” risk premium, and a “bear” market, with low returns and high volatility, or a low “variance-adjusted” risk premium (Ang and...
Economic conditions change stochastically between the two markets. We study the optimal investment problem for a small investor who maximizes logarithm utility over his terminal wealth. The investor can invest in a risky asset and a risk-free asset. He incurs proportional transaction costs for his trades.

Our analysis reveals that the investor may find it optimal to sell some risky assets to reduce the degree of leverage, but still keep a positive leverage ratio, or not to reduce leverage at all in the “bear” market, even if the variance-adjusted risk premium becomes negative. This result is in sharp contrast to previous studies in the literature that assume constant investment opportunity set: leverage is optimal if and only if such variance-adjust risk premium is positive (Merton, 1969, 1971; Dai et al., 2009; Dai and Yi, 2009). We show that the investor may not deleverage at all in the “bear” market in this two-market economy, even though it is optimal for him to do so with only one market. Instead, the investor may keep his leverage position unchanged or reduce the degree of leverage, particularly so for illiquid assets. We further show that a lower borrowing cost in the “bear” market encourages stock holdings and prevents sell offs.

The study of portfolio selection with transaction costs in a constant economic environment has a long history. Magill and Constantinides (1976) first introduce proportional transaction costs into Merton’s model. Constantinides (1979, 1986) considers an infinite horizon problem where the investor aims to maximize discounted utility of intermediate consumption in a constant market. Davis and Norman (1990) present a rigorous mathematical formulation of free boundary problem and then study the properties of the free boundary that stands for the optimal strategy, and Shreve and Soner (1994) make use of a viscosity solution approach to entirely characterize the behaviors of the free boundary for an infinite horizon problem.

Finite horizon portfolio selection with transaction costs has not been solved until recently. The no transaction region becomes time dependent and thus the investor’s problem becomes much harder to solve. Liu and Loewenstein (2002) examine the optimal strategy of a pure investment problem, still in a constant market, by virtue of a sequence of analytical approximation solutions. In this paper, we are able to provide a complete mathematical characterization of the time-dependent boundaries of the transaction regions in the face of changing economic conditions.

Jang et al. (2007) consider an infinite horizon problem in a bull–bear switching market and aim to explain the puzzle of liquidity premium. Liu and Loewenstein (2009) study a jump diffusion model with regime switching, and aim to address the “flight to quality” phenomena after market crashes. In the present paper, we consider a finite horizon problem and attempt to address the issue of leverage management through a thorough characterization of an optimal investment policy. Our results shed light on corporate and consumer deleveraging decisions in changing economic environment. The time-dependent investment model considered allows us to provide a clean mathematical characterization of the investor’s rebalancing strategy against the underlying parameters of the economy.

The remainder of the paper is organized as follows. Section 2 describes the investor’s optimal investment problem with transaction cost and regime switching. Section 3 is devoted to the characterization of the optimal strategy and leverage management. Section 4 concludes the paper.

2. Formulation of the model

2.1. The investor’s problem

Assume that an investor can trade two assets, a riskless asset (“the bank account”) and a risky asset (“the stock”). The price dynamics are given by

\[ dP_t = r(\epsilon_t) P_t \, dt, \]

\[ dQ_t = Q_t [ \mu(\epsilon_t) \, dt + \sigma(\epsilon_t) \, dB_t ]. \]

Here \( \epsilon_t \in \{1, 2\} \) denotes the changing investment opportunity set that switches between two regimes—a “bull” market and a “bear” market, which is governed by a two-state Markov chain with generators

\[
\begin{pmatrix}
  k_1 & -k_1 \\
  -k_2 & k_2
\end{pmatrix}
\]


\[ \frac{dP_t}{P_t} = r(\epsilon_t) dt + \sigma(\epsilon_t) dB_t, \]

\[ \frac{dQ_t}{Q_t} = \mu(\epsilon_t) dt + \sigma(\epsilon_t) dB_t. \]

Here \( \epsilon_t \in \{1, 2\} \) denotes the changing investment opportunity set that switches between two regimes—a “bull” market and a “bear” market, which is governed by a two-state Markov chain with generators

\[
\begin{pmatrix}
  k_1 & -k_1 \\
  -k_2 & k_2
\end{pmatrix}
\]

3 Merton (1969, 1971) considers portfolio selection in a constant market without transaction costs and concluded that leverage is optimal if and only if the variance-adjusted risk premium is positive (for both finite horizon and infinite horizon case). Dai and Yi (2009) consider a pure investment problem in a constant market with transaction costs and finite horizon and concluded that leverage is optimal if and only if the variance-adjusted risk premium is positive when the time to maturity is sufficiently long. The result is further extended to the case of consumption in Dai et al. (2009).


5 Dai and Yi (2009) consider the same problem and derive an equivalent variational inequality through which the optimal strategy can be entirely characterized. Dai et al. (2009, 2010a) further extend it to the case of consumption and the mean-variance framework, respectively. Numerical methods and solutions are provided by Gennotte and Jung (1994), Dai and Zhong (2010), etc.
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