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The design of equity-indexed annuities

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ABSTRACT

There is a rich variety of tailored investment products available to the retail investor in every developed economy. These contracts combine upside participation in bull markets with downside protection in bear markets. Examples include equity-linked contracts and other types of structured products. This paper analyzes these contracts from the investor's perspective rather than the issuer's using concepts and tools from financial economics. We analyze and critique their current design and examine their valuation from the investor's perspective. We propose a generalization of the conventional design that has some interesting features. The generalized contract specifications are obtained by assuming that the investor wishes to maximize end of period expected utility of wealth subject to certain constraints. The first constraint is a guaranteed minimum rate of return which is a common feature of conventional contracts. The second constraint is new. It provides the investor with the opportunity to outperform a benchmark portfolio with some probability. We present the explicit form of the optimal contract assuming both constraints apply and we illustrate the nature of the solution using specific examples. The paper focusses on equity-indexed annuities as a representative type of such contracts but our approach is applicable to other types of equity-linked contracts and structured products.

1. Introduction

This paper discusses and critiques equity-linked contracts. These products are important since they constitute a popular class of investment contracts for retail investors. We analyze them from the investor's perspective. This viewpoint contrasts with much of the actuarial literature which is based on the issuer's perspective and focusses on the pricing, hedging and risk management of these contracts. We discuss the optimality of the conventional design of these contracts from the consumer's viewpoint and note that in general the design is inefficient. We propose a new type of product which we call the *generalized contract*. Under a generalized contract an investor has the opportunity to beat a benchmark index with some probability and also has a guaranteed minimum rate of return, irrespective of market performance. We give the optimal contract design of this generalized contract and explore its properties using specific examples.

At this stage, a brief description of equity-indexed annuities may be helpful. In a single premium contract, the customer(investor) pays an initial amount (the premium) to the insurer. Suppose that the contract matures in say five years. At maturity the payoff is based on the performance of some reference index which could be, for example, a stock market index. The contract participates in the gains (if any) in the reference portfolio during this period. The detailed arrangements of how this participation is calculated vary, but usually the participation has some call option features. In addition these contracts provide a floor of protection if the market does poorly. For example the guaranteed floor may be a return of the initial investment which means that the investor has an embedded put option. In some contracts there is a maximum or cap on the investor's return. The cap can limit the return over the life of the contract or it may operate period by period. In some circumstances the investor has an early redemption option to cancel or surrender the contract. In this paper we use the equity-indexed annuity (EIA) as a representative example of this class of contracts in our analysis. These represent a very popular[1] class of structured products sold by insurance companies in USA with a rich variety of design features.

At first sight the EIA appears to deliver the best of both worlds: an investment that goes up when the stock market goes up but provides a guaranteed floor if the market collapse.

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[1] According to the industry group LIMRA, sales of EIA were around 24 billion US dollars in 2004, up from $ 14.4 billion in 2003, and the sales volume is even bigger in 2005.

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However, these contracts have some features that may reduce their attractiveness to retail investor. First, many contracts have high initial commissions payable to the agent who sells the contract to the investor. Second, the contract design is often complicated making it difficult for the consumer to understand the product. Third the design may not really suit the consumer's needs. One of the main aims of this paper is to address this last feature. Specifically, we discuss how to design a contract that is optimal for the customers under certain assumptions. Our proposed contract is generally more complicated than existing products. However this optimal contract provides a useful benchmark.

There is one aspect of our analysis that deserves some discussion. This arises from our use of a framework where financial institutions such as insurance companies exist and the extent to which we can simultaneously use the assumptions of no-arbitrage and complete markets in the same framework. Strictly speaking, in a world of perfect information, frictionless markets and a complete set of marketed securities there is no role for financial intermediaries. In this world consumers can use the existing securities to maximize their expected utility. However, in practice individual consumers face significant transaction costs and informational costs if they want to trade directly to form their optimal portfolio. Merton[2] has discussed the role of financial intermediaries in this setting and noted that individuals use financial institutions which face lower transaction costs to perform these functions. Indeed as Merton and Bodie (2005) note

· · · in a modern well developed financial system the lowest cost transactor may have marginal trading costs close to zero and can trade almost continuously. Our approach uses a similar perspective to Merton's. Financial institutions sell contracts to their customers. The institutions are able to replicate payoffs with essentially no transaction costs. However an individual customer may be quite willing to pay a higher premium for the contract than its strict no-arbitrage value because he(she) faces significant transaction costs.

There is an extensive actuarial literature on the pricing, hedging and risk management of these contracts. See for example Biffis and Millossovich (2006), Nielsen (2006), Hardy (2003) and Tiong (2000). This approach is termed the *fair premium* approach and it can be traced back to Black and Scholes (1973), Merton (1973), Brennan and Schwartz (1976), Boyle and Schwartz (1977). Since the present value of the claim is often calculated by employing the no-arbitrage principle, we argue that the (no-arbitrage) present value of the payoff is just the first step in investigating this kind of contract from the consumer's perspective. This follows from the discussion in the previous paragraph. The issuer will be able to charge the customer more than the no-arbitrage value of the contract.

The aim of this paper is to critique the conventional structure and design of EIAs and suggest a possible generalization. In the spirit of the papers of Arrow (1974) and Raviv (1979), we investigate the optimal design of an EIA under the expected utility framework.[3] The expected utility framework is natural for this analysis because the contract is an investment vehicle and it is not reasonable to assume that investors are risk-neutral. Moore and Young (2005) also use the expected utility approach in the context of the design of a perpetual equity-indexed annuity.

In a conventional EIA, both the guaranteed return and the equity-indexed return are incorporated into one guaranteed payoff. Therefore, the optimal design problem for an EIA becomes a standard optimal portfolio selection problem (See Cox and Huang (1989), Merton (1971) and Pliska (1986)) subject to a minimal guaranteed on the terminal wealth. The optimal payoff under the EIA is the optimal terminal wealth in this optimal portfolio selection problem, and it is derived explicitly in Theorem 3.1 of Section 3.4. We show that the current payoff structure of the conventional EIA is not optimal for most investors.[4] We are also able to investigate the impact of different investor risk preferences on the return to the investor.

In the second part of the paper we introduce a new type of equity-linked product which we call a generalized equity index annuity. Boyle and Tian (in press) have derived the explicit solution for the optimal contract in this case under fairly general conditions. Here we develop the analysis using the equity-indexed annuity as the prototype but our results hold for other types of structured products with appropriate modifications. We assume that the preferred contract will maximize the investor's expected utility of terminal wealth subject to two constraints. The first constraint is that there is a minimum guaranteed rate of return on the contract. This constraint is a very pervasive feature in these types of tailored retail products. The second constraint gives the investor an opportunity to outperform some benchmark with a certain probability α. We provide justification for this type of constraint later but, for now, we note that in the special case when $\alpha = 1$, this aspect of the contract corresponds to the equity participation feature of existing contracts. Since we do not account for the mortality and surrender features, the contracts considered in this paper are very similar to structured products[5] which are sold by banks.

Under some plausible assumptions we are able to solve for the optimal contract design of the generalized equity-indexed annuity. The explicit solution depends on the investor's utility function, the chosen benchmark and the confidence level as well as certain capital market parameters. The optimal payoff at maturity displays an interesting dependence on the level of the underlying index. In general there will be discontinuities and payoff need not be an increasing function of the index. The discontinuity arises from the existence of the probabilistic constraint and Basak and Shapiro (2001) document the same type of behavior in their study of VaR.

The rest of the paper is organized as follows. In Section 2, we discuss in some detail a popular type of EIA known as a point-to-point EIA. We examine the valuation and design of this contract from the investor's perspective. We extend the analysis to other types of EIAs in Section 3. Section 4 describes our proposals for a generalized EIA. We also exhibit and discuss the solution to the optimal design of the generalized contract. Section 5 describes the explicit construction of the optimal design for a specific type of generalized contract and gives several numerical examples to help describe its main features. Section 6 concludes the paper. Some technical formulae are given in the Appendices A and B.

[2] Merton and Bodie (2005) state: "But in the presence of substantial informational and transactional costs it is not realistic to posit that the only process for individuals to establish their optimal portfolios is to trade each separate security for themselves directly in the markets. Instead individuals are likely to turn to financial organizations such as mutual funds and pension funds that can provide pooled portfolio management services at a much lower cost than individuals can provide for themselves".

[3] There is also some research on the optimal design without using the utility framework. See Doherty and Eeckhoudt (1995), Doherty and Schlesinger (1983), Gollier and Schlesinger (1996) and Schlesinger (1997).

[4] This is not surprising that these insurance contracts are not optimal or Pareto-efficient. For instance, Brennan (1993) developed a nice analysis of the non-optimality of some insurance contracts. Since we explore the investor's perspective by including risk preferences, our analysis is different from Brennan's (1993).

[5] Generally speaking, a structured product is often based on a complicated underlying index, while an EIA may have a complicated formula for computing the investor's return. In USA structured products are registered under the Securities Act as securities and most EIAs are registered as insurance products. See Francis et al. (2000) for a discussion of structured products.