Electrical Power and Energy Systems 32 (2010) 271–280

Contents lists available at ScienceDirect

Electrical Power and Energy Systems

journal homepage: www.elsevier.com/locate/ijepes

A backward sweep method for power flow solution in distribution networks

A. Augugliaro, L. Dusonchet, S. Favuzza *, M.G. Ippolito, E. Riva Sanseverino

Dipartimento di Ingegneria Elettrica, Elettronica e delle Telecomunicazioni, Università di Palermo, 90128 Palermo, Italy

ARTICLE INFO

Article history:
Received 17 December 2007
Received in revised form 8 September 2009
Accepted 25 September 2009

Keywords:
Backward/forward method
Radial and meshed distribution networks
PV nodes
Power flow

ABSTRACT

A methodology for the analysis of radial or weakly meshed distribution systems supplying voltage dependent loads is here developed. The solution process is iterative and, at each step, loads are simulated by means of impedances. Therefore, at each iteration, it is necessary to solve a network made up only of impedances; for this kind of network, all the voltages and currents can be expressed as linear functions of a single unknown current (in radial systems) or of two unknown currents for each independent mesh (for meshed systems). The methodology has been called "backward" since the unique equation, in case of radial network, and the linear system of equations, in case of meshed network, in which such unknown currents appear can be determined by starting from the ending nodes of the radial system, or of the radialized network (obtained by means of cuts in meshed networks). After a brief presentation of the b/f method, which is currently the most commonly used technique for solving distribution networks, the solution methodology is detailed both for radial and for meshed systems. Then, the way in which PV nodes can be considered is also described.

Finally, the results obtained in the solution of some networks already studied in the literature are presented with other methods, in order to compare their performances.

The applications show the efficiency of the proposed methodology in solving distribution networks with many meshes and PV nodes.

1. Introduction

The method currently adopted for the analysis of radial distribution systems is the backward/forward method (b/f) [1–12], which, in only one iteration for constant current loads, or in more than one iteration for other types of loads (constant power, mixed, etc.) finds the solution.

It is well known that there exist three main variants of the b/f method that differ from each other based on the type of electric quantities that at each iteration, starting from the terminal nodes and going up to the source node (backward sweep), are calculated:

(i) the current summation method, in which the branch currents are evaluated;
(ii) the power summation method, in which the power flows in the branches are evaluated;
(iii) the admittance summation method, in which, node by node, the driving point admittances are evaluated.

In other terms, the three variants of the b/f method simulate the loads within each iteration, with a constant current, a constant power and a constant admittance model. In the forward phase, the three variants are identical since, based on quantities calculated in the backward phase, the bus voltages are calculated starting from the source node and going towards the ending nodes. Voltages are then used to update, based on the dependency of loads on the voltage, the quantities used in the backward sweep in order to proceed to another iteration.

The process stops when a convergence criterion is verified. If the network is meshed, the most commonly adopted solution process is that of radializing the network by means of a certain number of cuts [13–16]. For each couple of nodes, created by each cut, two equal and opposite currents are injected, the value of which is determined by imposing the condition that the voltage difference between the two cut nodes goes to zero. This is the compensation currents method [17]; it uses a reduced Thévenin impedance matrix and a vector of known terms that are the open circuit voltages between the cut nodes. The latter are determined, for the radialized network, at the end of a forward phase. Since the condition defined at the cut nodes is linear (equality of the voltages) in the unknowns (the compensation currents), the system to be solved is also linear and its resolution requires the inversion of the reduced Thévenin impedance matrix. The latter is composed of terms that do not depend on bus voltages; therefore, it is enough to invert it only once and keep the coefficients of the inverse matrix in order to use them in the different iterations.

* Corresponding author. Tel.: +39 0916615221; fax: +39 091488452.
E-mail address: favuzza@dieet.unipa.it (S. Favuzza).

0142-0615/$ - see front matter © 2009 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijepes.2009.09.007

The compensation currents method is also used to solve a network with PV nodes [18–21]; in this case, fictitious meshes are added; these are obtained by connecting a null impedance branch between the PV node and the node taken as the reference for voltage at the source node. In such a branch, an ideal voltage generator is inserted, the magnitude of which is equal to the imposed voltage at the PV node. The solution of a network having real and fictitious meshes associated to PV nodes is carried out with the above-described method, executing cuts in all the meshes in order to radialize the network. The cuts in the fictitious meshes are executed so that the two cut nodes are: one, the PV node of the network; the other, the pole of the ideal voltage generator. The construction of the reduced Thévenin impedance matrix is carried out based on all the meshes, both real and fictitious.

A wide review and a comparison study are presented in [22], where various distribution system load flow algorithms, based on the forward/backward sweeps, are reviewed, and their convergence ability is quantitatively evaluated for different loading conditions, R/X ratios, and substation voltage levels; moreover, the effect of static load modeling on the convergence characteristics of algorithms is investigated.

The analysis methodology set-up here is based on the solution, at each iteration, of a radial or radialized network made up of series and shunt impedances and supplied by one point. The series impedances are those of the lines, while the shunt impedances are: the capacitances of the lines (concentrated at the two ends); the capacitors for the reactive power compensation; and the load equivalents. The load impedances are evaluated at the beginning of each iteration, based on the rated values of the power of the loads, on the dependency of the loads on the voltage, and on the loads' bus voltages (such values are fixed at the first iteration, and are calculated in the subsequent iterations).

The simulation of the loads by means of impedances is used in [23] to solve radial or meshed distribution systems; the unknowns of the problem are, for radial systems, all the loads currents; meshed systems are turned into radial by means of cuts and, in this case, the unknowns are all the loading currents and the cut currents of the meshes. The solution of the network goes through the construction of an impedance matrix linking the unknown currents with the source node voltage; all the unknown currents are determined by solving a linear system of equations.

Differently from the methodology developed in [23], the technique here set up, at each iteration allows, in the case of radial systems, to have only one unknown current in the entire system, the value of which can be obtained based on the value of the imposed voltage at the source node of the network. For meshed systems, the unknown currents, the number of which is twice the number of independent meshes, can be gained by solving a linear system of equations obtained by starting from the radialized network and related to the source node of the network and to the nodes from which branches belonging to the meshes spread out.

Differently from the compensation currents method, in which the meshes are considered after having solved the radialized system, in the methodology here set up the unknown currents of the meshed system are solved all together; in this way, it is not necessary to execute the correction of the bus voltages following the interesting and efficient technique proposed by Rajicic et al. in [19] and set up in the aim of limiting the drawbacks of the compensation currents method.

The other quantities of the network, bus voltages and branch currents can be obtained directly from the values of the unknowns and as a linear combination of them. Differently from the b/f method, the calculation of the voltages is not carried out sequentially starting from the source node and going towards the ending nodes; at each iteration, each voltage can be evaluated independently from the others. The methodology is "backward" since the equations with the unknown currents (for radial networks) and the linear system with the unknown currents (for meshed networks) are obtained starting from the ending nodes of the radial system, or from the cut nodes in radialized systems.

The applications show the efficiency of the proposed methodology in solving distribution networks in complex situations, namely in networks with many meshes and fixed voltage nodes. These features are particularly favorable in optimization problems solved by means of the analysis of many possible solutions as, for example, the reactive power flows compensation ([24]) or the service restoration [25].

2. General analysis methodology

Within the proposed methodology, at each iteration, the loads at the nodes are modeled by means of impedances calculated based on the bus power and voltage. So, for each iteration, the main problem is how to solve efficiently a network made up of series and shunt impedances; to this aim, a methodology to solve radial and meshed networks (presented in the following sections) has been set up. Once the bus voltages have been determined, they are compared with those that have been used to evaluate the load impedances. If the error is below a prefixed margin, the iterative process stops, otherwise another iteration is started.

2.1. Radial networks solution

Consider the network in Fig. 1 made up of a single feeder supplying N loads; node 0 is the source node with constant voltage equal to $\boldsymbol{V}_0$ (hereafter, bold letters indicate phasors or complex quantities, and roman letters real quantities).

Under the assumption that loads are impedances, the network can be represented by the cascade of series and shunt impedances shown in Fig. 2; the series impedances, $\boldsymbol{Z}_{\boldsymbol{ser},\boldsymbol{i}}$, include the line impedances, while the load impedances, the line capacitances and the capacitors for reactive power compensation are included in the shunt impedances, $\boldsymbol{Z}_{\boldsymbol{sh},i}$. Calling $\boldsymbol{I}_{\boldsymbol{Zsh},N}$ the unknown current in the last shunt impedance, $\boldsymbol{Z}_{\boldsymbol{sh},N}$, starting from the terminal nodes and going up to the source node, all the bus voltages ($\boldsymbol{V}_{\boldsymbol{i}}$), all the load currents ($\boldsymbol{I}_{\boldsymbol{L},\boldsymbol{i}}$) and all the branch currents ($\boldsymbol{I}_{\boldsymbol{b},\boldsymbol{k}}$) can be calculated. These quantities are proportional to the current $\boldsymbol{I}_{\boldsymbol{Zsh},N}$ following relations such as:

$$\boldsymbol{V}_{\boldsymbol{i}} = \boldsymbol{H}_{\boldsymbol{V}}(i,N) \cdot \boldsymbol{I}_{\boldsymbol{Zsh},N} \tag{1}$$

$$\boldsymbol{I}_{\boldsymbol{L},\boldsymbol{i}} = \boldsymbol{H}_{\boldsymbol{L}}(i,N) \cdot \boldsymbol{I}_{\boldsymbol{Zsh},N} \tag{2}$$

$$\boldsymbol{I}_{\boldsymbol{b},\boldsymbol{k}} = \boldsymbol{H}_{\boldsymbol{I_b}}(k,N) \cdot \boldsymbol{I}_{\boldsymbol{Zsh},N} \tag{3}$$

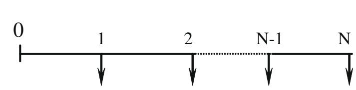

Fig. 1. Main feeder.

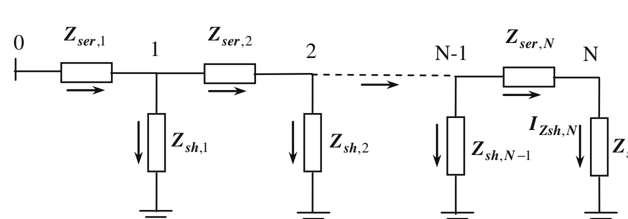

Fig. 2. Scheme of the main feeder shown in Fig. 1 with series and shunt impedances.