Semi-Markov Processes for Power System Reliability Assessment With Application to Uninterruptible Power Supply

Antonio Pievatolo, Enrico Tironi, and Ivan Valadè

***Abstract*—We propose a state space model for electrical power systems made by independent semi-Markov components, in which restoration times can have a nonexponential distribution, thus obtaining a more realistic reliability characterization, especially regarding the outage duration distribution. We also propose a model for an energy storage unit, assuming that the storage is fully charged when it begins to deliver power. An approximate analytical evaluation based on the minimal cut sets for the outage allows to surmount the shortcomings of the Monte Carlo approach. The application of the model for an uninterruptible power supply (UPS) system shows that the autonomy of the storage plays a key role, not only for the frequency of the load point voltage failures, but also for their duration distribution.**

***Index Terms*—Energy storage system, minimal cut sets, power system reliability, semi-Markov stochastic processes, UPS.**

NOMENCLATURE

C	Number of components in the system.
$X_c(t)$	State of component c at time t.
i, j, x_c	Different ways of indicating a state taken by component c.
N_c	Number of states that component c can take.
$P_c(i,j)$	Probabilities of the transition matrix P_c of the embedded Markov chain for component c.
$F_{c,i}(t)$	Distribution function of the sojourn time of the component c in the state i.
$F_{c,ij}(t)$	Distribution function of the sojourn time of the component c in the state i, given that the next state will be j.
D_{c,x_c}	Amount of time spent by component c in state x_c from the transition to it to the next transition.
$S(t)$	State of the system at time t.
$s=(x_1,\ldots x_C)$	Fixed state of the system.
k, h, u	Indexes of the generic minimal cut set of the system.
Ω	Set of the minimal cut sets of the system for a given load point.
$\bullet^{(h)}$	Quantity referring to the minimal cut set h.
$\Omega^{(h)}$	States of the failed components in the minimal cut set h.

Manuscript received October 20, 2003.

A. Pievatolo and I. Valadè are with the I.M.A.T.I., National Research Council, Milano 20133, Italy (e-mail: marco@mi.imati.cnr.it; ivan@mi.imati.cnr.it).

E. Tironi is with the Electrical Engineering Department, Politecnico di Milano, Milano 20133, Italy (e-mail: enrico.tironi@polimi.it).

Digital Object Identifier 10.1109/TPWRS.2004.826756

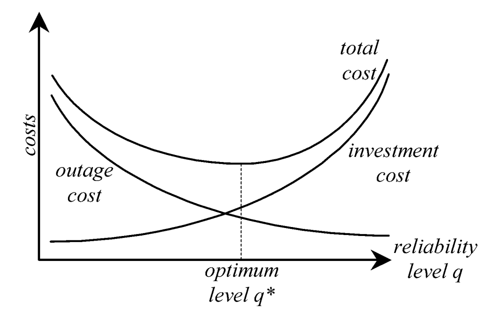

Fig. 1. Choice of the optimum reliability level minimizing the total cost.

I. INTRODUCTION

RELIABILITY is the measure that describes the ability of a system to perform its intended function. Reliability levels are interdependent with economics since increased reliability is obtained through increased investments but it also allows the consumers to reduce their outage costs. In order to carry out objective cost-benefit studies, both the previous economical aspects are important: the optimum reliability level is determined by minimizing the total cost, as reported in Fig. 1.

This paper examines some probabilistic tools that can be used to deal with what is generally known as "reliability worth," that is, the benefit derived by the investments aimed at increasing the reliability thanks to the reduction of the outage cost.

The reliability assessment is generally concerned with average indexes, such as the mean time between failure (MTBF) and the mean time to restoration (MTTR). Also, the probability distribution from which these mean values are derived is useful to more accurately estimate the outage cost of the users with very nonlinear costing function. As shown in [1] through simulations, the duration distribution of the restoration activities heavily affects the probability distribution of the outage duration, so mathematical models admitting distributions other than the negative exponential are necessary.

In [2], a state-space model was employed for the analytical calculation of the MTBF and the MTTR for a system made up of independent semi-Markov components (see [3] for the general theory about these processes): the model was applied to the reliability assessment of an uninterruptible power supply (UPS). The same model could also be useful for the electrical power distribution system when it is assumed that its behavior can be obtained from that of each component computed separately. In fact, in most cases, the complexity of the problem does not allow one to apply the analytical models embedding both dependencies and nonexponential distributions ([4], Sec. I.C), as it is necessary to solve the state transition diagram of the whole system. To the knowledge of the authors, in the field of the power system analysis, these analytical models, such as the auxiliary variable or the device of stage methods, have been applied only to a very small set of apparatuses [5]–[7]. Furthermore, some dependencies can be treated by the model used in this paper by grouping in a single component the dependent devices. This is especially true for the common-cause failures in redundant devices.

The calculation of the approximate duration distribution of a load point voltage failure via the minimal cut sets, as reported in [8], improves both [2], where this distribution is determined by simulation, and [9] in which the distribution of a single system state only is computed. The list of the minimal cut sets can be obtained through the FMEA analysis of the power distribution system, as suggested in IEEE Standard 493 [10], or from the fault tree relating to the load point being studied [11]. Today, this methodology is applied in power distribution system reliability studies [12], [13], even if for large systems, the difficulty of identifying all of the minimal cut sets compel to consider the low-order ones only.

Distributed energy storage systems are beginning to be installed in the recent power distribution systems, as it has been proposed for the Flexible, Reliable and Intelligent Electrical Energy Delivery System (FRIENDS), [14]. From one point of view, the energy storage systems allow the use of new energy resources, and especially the renewable ones; from the other, they are used to mitigate some aspects of the power quality problem. The aspect of voltage stability, which includes voltage sags, requires a small amount of energy to be stored, while the compensation of the continuity of supplying power requires a larger amount of energy stored. Therefore, an effort has been made for the explicit modeling of the storage unit, whose state of charge depends on the stochastic stories of all the semi-Markov independent components. This is different from [15] where the storage is considered as embedded within a single semi-Markov model.

After a brief summary of the results of [2] (Section II-A), special attention is paid to the calculation of the distribution of the outage time at any load point. This is done by combining the duration distribution of the minimal cut sets for the load point outage, both when an unlimited storage is available (Section II-B) and when it is not (Section III). The method is then applied to an UPS with a limited storage (Section IV).

II. Stochastic Model Made up by Independent Semi-Markov Components

In this section, the energy storage autonomy is considered unlimited and, therefore, the reliability quantities have the symbol ∞ as apex.

The load point voltage is said to be in the failure state when it is out of the rated limits, otherwise it is in the functioning state. This state results from a combination of the states of all the components that make up the electric power system: the relationship between the state of the components and of the load point can be described by a fault tree, as will be shown in Section IV.

The following hypotheses have been assumed:

- stochastic independence among the power system components;
- the repair of a component starts as soon as the fault has occurred. The repair time includes the technician's travelling time, fault identification and repair, and putting the component back into service.

The stochastic model is studied under steady-state conditions.

A. Steady-State Average Reliability Indexes of a Load Point

The stochastic behavior of the power system can be described by first modeling separately each component c through a semi-Markov process, and then using the fact that the components change states independently of each other.

For a semi-Markov process, the transitions among the states may be thought of as taking place in two stages: first of all, when the state i is entered, the next state is chosen according to the transition matrix P_c; then, given that the next state is j, the sojourn time in the state i has a distribution $F_{c,ij}(t)$, taken absolutely continuous. For the components that make up the power system, we have a positive recurrent semi-Markov process for which the steady-state probability $\Pr_c(i)$ of component c being in state i can be calculated starting from the invariant probability distribution $(\pi_{c,1},\ldots,\pi_{c,N_c})$ of P_c

$$\Pr_c(i) = \lim_{t\to\infty} \Pr\left(X_c(t)=i \mid X_c(0)=j\right) = \frac{\pi_{c,i}\cdot\mu_{c,i}}{\sum_{k=1}^{N_c}\pi_{c,k}\cdot\mu_{c,k}}, \quad \forall\, j \tag{1}$$

where $\mu_{c,i}$ is the mean of the sojourn time of the component c in the state i.

In order to carry out the reliability analysis, it is necessary to determine the steady-state frequency of passing from a set of states, named F, to another, named B. As is shown in [2], this frequency can be calculated as

$$Fr_{F\to B} = \lim_{t\to\infty}\lim_{\Delta\to 0}\frac{\Pr\left(S(t)\in F \cap S(t+\Delta)\in B\right)}{\Delta} = \sum_{s\in F}\Pr(s)\cdot\left(\sum_{c=1}^{C}\sum_{\substack{x'_c=1\\ s(x'_c)\in B}}^{N_c}\frac{P_c\left(x_c,x'_c\right)}{\mu_{c,x_c}}\right) \tag{2}$$

where $s\left(x'_c\right)$ denotes the power system state s where only component c changes state from x_c to x'_c, because the probability of having two or more component changing states in $[t, t+\Delta]$ is negligible due to the absolute continuity of the duration distributions of the sojourn times.

If F is the set of those s states of the power system that cause the examined load point voltage to be in the failure state, and B is the set of all the other states, the average load point failure rate (λ_{LP}^{∞}) can be calculated through (2). The average load point availability, which is the mean proportion of time that the load point voltage is in the rated limits, can be calculated from (1) as

$$A_{LP}^{\infty} = \sum_{s\in B}\Pr(s) = \sum_{s\in B}\left[\prod_{c=1}^{C}\Pr_c\left(x_c\right)\right], \quad s=(x_1,\ldots,x_C). \tag{3}$$