A Discrete Fourier Transform-Based Adaptive Mimic Phasor Estimator for Distance Relaying Applications

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***Abstract*—In protection relaying schemes, the digital filter unit plays the essential roles to calculate the accurate phasor. However, while the fault current contains plentiful decaying dc component, the over-reach of distance protection will cause sever problem. This work develops an adaptive mimic phasor estimator to remove the decaying dc oscillation between voltage and current and obtains the accurate apparent impedance. First, a discrete Fourier transform-based mimic phasor estimator is developed. Then, an adaptive scheme is proposed to obtain the decaying time constant. Unlike the fixed decaying dc time constant used in a digital mimic filter, the proposed algorithm adopts the transmission-line parameters information hiding in the voltage and current measurements to adaptively approximate the decaying dc time constant to the accurate value. Thus, the estimation error in the mimic filter due to the time constant mismatch can be eliminated. Both full-cycle and half-cycle versions are developed in this work. Simulations results illustrate the effectiveness of this new algorithm for distance relaying applications.**

***Index Terms*—Adaptive mimic phasor estimator, decaying dc component, discrete Fourier transform, distance relaying, phasor.**

I. INTRODUCTION

IN PROTECTION relaying schemes, the digital filter technologies have played the essential roles. Usually, the most widely used algorithm is the discrete Fourier transform (DFT) [1]. When the measurements only contain fundamental frequency and integer harmonic frequency components, full-cycle DFT only needs one-cycle samples to obtain the accurate fundamental phasor. However, in most cases, the fault currents contain a decaying dc component. Since the noninteger harmonic components are in decaying dc, the DFT algorithm does not have the ability to filter out the decaying dc component.

Many papers [2]–[5] have been proposed to remove the decaying dc contained in phasor. In [2], Benmouyal has proposed a digital mimic filtering technique to remove the decaying dc component. The decaying dc time constant is first given by transmission-line analysis. Then, the decaying dc is removed by the digital mimic circuit equation. In [3] and [4], Gu and Yang have proposed DFT-based algorithms to completely remove the decaying dc component. Those algorithms use three continuous DFT results to calculate the unknown parameters of the decaying dc component. Thus, the time constant and magnitude of the decaying dc component can be exactly obtained, and the decaying dc component can be removed. In [5], Sidhu has also proposed a DFT-based filtering technique to remove the decaying dc component. This algorithm uses the simultaneous equations developed by a different harmonic basis to obtain the unknown parameters of a decaying dc component. Thus, the decaying dc component can be quickly removed by one- or half-cycle samples.

The above algorithms can be categorized into two groups according to the way the decaying dc time constant is obtained.

1) By waveform analyzing: As proposed in [3]–[5], the design of a digital filter is based on analyzing the signal waveform. First, the decaying dc component is assumed to be involved in measurement. Then, the exact solution of magnitude and time constant of decaying dc component are determined by digital filtering process. Finally, the accurate phasor can be obtained by removing the decaying dc component.
2) By transmission-line analyzing: As proposed in [2], the design of the mimic filter is based on analyzing the transmission-line parameter. First, a proper decaying dc time constant is found. Then, the decaying dc time constant is given as a constant to the digital mimic filter. Finally, the decaying dc component can be removed by mimic circuit equations.

If the decaying dc component does exist, the waveform analyzing method can obtain very accurate results. However, for the normal signal that the decaying dc component does not involve, there needs to be some extra scheme for judgment and tuning the filtering process. Otherwise, the filtering result may become divergent. On the other hand, the transmission-line analyzing method is more robust. The mimic filter can work properly for both the normal signal and the signal involving the decaying dc component cases. If the time constant given to the mimic filter is very close to the actual value, the decaying dc can be removed completely from measurement. However, if the time constant specified to the mimic filter is far from the correct value, the mismatch time constant will cause the non-negligible error.

This work develops an adaptive mimic phasor estimator (AMPE) to remove the decaying dc between voltage and current phasors and obtains the accurate apparent impedance. The block diagram of the proposed estimator is depicted in Fig. 1. First, a DFT-based mimic phasor estimator is developed. Then, an adaptive scheme is proposed to obtain the decaying time constant. Unlike the fixed decaying dc time constant used in the digital mimic filter, the proposed algorithm adopts the transmission-line parameters information hiding in the voltage and current measurements to adaptively approximate the decaying dc time constant to the accurate value. Thus, the estimation error in the mimic filter due to the time constant mismatch can

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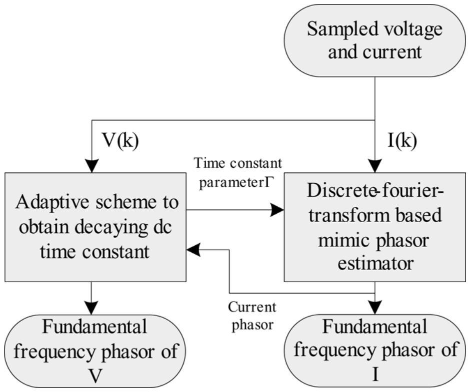

Fig. 1. Block diagram of the adaptive mimic phasor estimator.

be eliminated. Meanwhile, the proposed technique is similar to the mimic filter. It means that the filtering process can work properly for both the normal signal and the signal involving decaying dc component cases.

The rest of this paper is organized as follows. Section II presents the fundamental theory of DFT. Section III presents the DFT-based mimic phasor estimation algorithm. Section IV describes the adaptive scheme to obtain the decaying dc time constant. Section V presents some simulation results via some test signals and a fault transmission lines modeled by MATLAB [6]. Then, Section VI discusses some special phenomena from the simulation studies in detail. Conclusions are finally made in Section VII.

II. DFT

Consider a period T and continuous signal x(t) which contains constant dc and N/2 order harmonics. The signal x(t) is sampled for discrete computation. The sampling frequency= N time of fundamental frequency, and the sampling period $\Delta \mathrm{t} = \mathrm{T}/\mathrm{N}$. x(t) and the kth sample x(k) are represented as follows:

$$\mathrm{x(t)} = \mathrm{A_0} + \sum_{\mathrm{n=1}}^{\mathrm{N/2}} \mathrm{A_n} \cos(\mathrm{n}\omega \mathrm{t} + \theta_\mathrm{n}) \tag{1}$$

$$\mathrm{x(k)} = \mathrm{A_0} + \sum_{\mathrm{n=1}}^{\mathrm{N/2}} \mathrm{A_n} \cos\left(\frac{2\mathrm{nk}\pi}{\mathrm{N}} + \theta_\mathrm{n}\right) \tag{2}$$

where $\omega = 2\pi/\mathrm{T}$ and θ_n is the phase angle of nth harmonic.

The fundamental phasor is calculated in the real part $\mathrm{X_r}$ and imaginary part $\mathrm{X_i}$, respectively

$$\begin{aligned} \mathrm{X_r(M)} &= \frac{2}{M} \sum_{\mathrm{k=1}}^{\mathrm{M}} \mathrm{x(k)} \cos\left(\frac{\mathrm{k}2\pi}{\mathrm{N}}\right) \\ &= \frac{2}{M} \sum_{\mathrm{k=1}}^{\mathrm{M}} \mathrm{A_1} \cos\left(\frac{2\mathrm{k}\pi}{\mathrm{N}} + \theta_1\right) \cos\left(\frac{\mathrm{k}2\pi}{\mathrm{N}}\right) \end{aligned} \tag{3}$$

$$\begin{aligned} \mathrm{X_i(M)} &= \frac{-2}{\mathrm{M}} \sum_{\mathrm{k=1}}^{\mathrm{M}} \mathrm{x(k)} \sin\left(\frac{\mathrm{k}2\pi}{\mathrm{N}}\right) \\ &= \frac{-2}{\mathrm{M}} \mathrm{M} \sum_{\mathrm{k=1}}^{\mathrm{k}} \mathrm{A_1} \cos\left(\frac{2k\pi}{N} + \theta_1\right) \sin\left(\frac{k2\pi}{\mathrm{N}}\right) \end{aligned} \tag{4}$$

where M is the window length of DFT, $\mathrm{M} = \mathrm{N}$ for full-cycle DFT computation, and $\mathrm{M} = \mathrm{N}/2$ for half-cycle DFT computation.

The final fundamental phasor can be represented in polar form as follows:

$$\mathrm{A_1} = \sqrt{\mathrm{X_r^2(M)} + \mathrm{X_i^2(M)}} \tag{5}$$

$$\theta_1 = \tan^{-1}(\mathrm{X_i(M)}/\mathrm{X_r(M)}). \tag{6}$$

III. DFT-BASED MIMIC PHASOR ESTIMATION ALGORITHM

When the signal x(t) only contains constant dc and integer harmonics, DFT only needs one-cycle samples to calculate the accurate fundamental phasor. However, the fault currents always contain the decaying dc components. Since the decaying dc component is composed of noninteger harmonics, the accurate phasor will be obtained when the decaying dc component decays to a very small value.

In this section, we develop a DFT-based mimic phasor estimator to remove the decaying dc component. First, we define the signal x(t) as follows:

$$\mathrm{x(t)} = \mathrm{A_1} \cos(\omega \mathrm{t} + \theta_1) + \mathrm{Be}^{-\mathrm{t}/\tau} \tag{7}$$

where B and τ are the magnitude and time constant of the decaying dc component, respectively. We take N samples one cycle, the kth sample signal x(k) is presented as follows:

$$\mathrm{x(k)} = \mathrm{A_1} \cos\left(\frac{2\mathrm{k}\pi}{\mathrm{N}} + \theta_1\right) + \mathrm{B}\Gamma^{-\mathrm{k}} \tag{8}$$

where the decaying parameter $\Gamma = \mathrm{e}^{\Delta \mathrm{t}/\tau}$.

A. FCDFT-Based Implementation

Utilizing the full-cycle DFT algorithm, the real part and imaginary part of the fundamental phasor $\mathrm{X_r}$ (M) and $\mathrm{X_i}$ (M) are written as follows:

$$\begin{aligned} &\mathrm{X_r(M)} \\ &\quad= \frac{2}{\mathrm{M}} \sum_{\mathrm{k=1}}^{\mathrm{M}} \left[\mathrm{A_1} \cos\left(\frac{2\mathrm{k}\pi}{\mathrm{N}} + \theta_1\right) + \mathrm{B}\Gamma^{-\mathrm{k}}\right] \cos\left(\frac{\mathrm{k}2\pi}{\mathrm{N}}\right) \\ &\quad= \frac{2}{\mathrm{M}} \sum_{\mathrm{k=1}}^{\mathrm{M}} \mathrm{A_1} \cos\left(\frac{2\mathrm{k}\pi}{\mathrm{N}} + \theta_1\right) \cos\left(\frac{\mathrm{k}2\pi}{\mathrm{N}}\right) \\ &\qquad+ \frac{2}{\mathrm{M}} \sum_{\mathrm{k=1}}^{\mathrm{M}} \mathrm{B}\Gamma^{-\mathrm{k}} \cos\left(\frac{\mathrm{k}2\pi}{\mathrm{N}}\right) \end{aligned} \tag{9}$$

$$\begin{aligned} &\mathrm{X_i(M)} \\ &\quad= -\frac{2}{\mathrm{M}} \sum_{\mathrm{k=1}}^{\mathrm{M}} \left[\mathrm{A_1} \cos\left(\frac{2\mathrm{k}\pi}{\mathrm{N}} + \theta_1\right) + B\Gamma^{-\mathrm{k}}\right] \sin\left(\frac{\mathrm{k}2\pi}{\mathrm{N}}\right) \\ &\quad= -\frac{2}{\mathrm{M}} \sum_{\mathrm{k=1}}^{\mathrm{M}} \mathrm{A_1} \cos\left(\frac{2\mathrm{k}\pi}{\mathrm{N}} + \theta_1\right) \sin\left(\frac{\mathrm{k}2\pi}{\mathrm{N}}\right) \\ &\qquad- \frac{2}{\mathrm{M}} \sum_{\mathrm{k=1}}^{\mathrm{M}} \mathrm{B}\Gamma^{-\mathrm{k}} \sin\left(\frac{\mathrm{k}2\pi}{\mathrm{N}}\right). \end{aligned} \tag{10}$$

For FCDFT computation, window length $\mathrm{M} =$ one cycle samples N.