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Nonlinear weighted measurement fusion Unscented Kalman Filter with asymptotic optimality

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ABSTRACT

For the general nonlinear systems, a universal weighted measurement fusion (WMF) algorithm is presented via the Taylor series expansion method. Based on the proposed fusion algorithm and the well-known Unscented Kalman Filter (UKF), the WMF-UKF is presented. It is proven that the proposed WMF-UKF asymptotically approaches to the centralized measurement fusion UKF (CMF-UKF) with the increase of the order of Taylor series expansion. So it has the asymptotical global optimality. We find that WMF-UKF has less computational cost than the CMF-UKF does with the increase of the number of sensors. Two examples are given to show the effectiveness of the proposed algorithms.

1. Introduction

In recent years, information fusion estimation has been widely investigated in target tracking, navigation and guidance, signal processing, and so on [2,21]. In estimation theory, we often fuse the information from different sensors which can provide more knowledge on the interested target in time and space. For instance, the infrared sensor provides accurate angle but poor range while radar provides accurate range but poor angle [22]. There are two ways to achieve the fusion: state fusion methods (distributed convex combination of local estimators) [3,5,10,20,27] and measurement fusion methods [11,23]. For linear systems, there are perfect theories of state fusion methods, such as the federated filter [5], the unified optimal linear fusion [20] and three kinds of distributed state fusion methods [27]. For nonlinear systems, optimal centralized measurement fusion and suboptimal distributed state fusion are used in many literatures [12,15]. However, to the best knowledge of the authors, there is no optimal or asymptotically optimal weighted measurement fusion method for the general nonlinear systems.

For the measurement fusion methods, there are three fundamental structures: sequential fusion (SF), centralized measurement fusion (CMF) (augmented measurement vector based fusion) [28] and weighted measurement fusion (WMF) (data compression fusion) [11,23]. It is well-known that CMF is optimal due to no information loss. However, the computational cost of CMF will increase with the increase of the number of sensors since it only combines the measurements from all sensors to form an augmented measurement with a higher dimension. Comparatively speaking, WMF weights all the measurements from individual sensors and then forms a fused or compressed measurement with a lower dimension [24]. For linear

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systems, it has been well-known that WMF is functionally equivalent to CMF under certain conditions [23]. So WMF is globally optimal and has the reduced computational cost [18,8]. So far, WMF for nonlinear systems are only used for extended Kalman filter (EKF). However, the accuracy of the estimators based on EKF and WMF is low because of the constraints of the first-order Taylor expansion of EKF itself. So, weighted measurement fusion methods for nonlinear systems still need more attention and further research.

Over the past decade, there are a lot of nonlinear filtering algorithms based on Bayesian estimation, including the Unscented Kalman Filter (UKF) [16], the sequence Monte Carlo (SMC) [9], the Markov Chain Monte Carlo (MCMC) [4], the particle filter (PF) [1,13], and so on. These algorithms are only used to deal with the single sensor nonlinear systems. However, few results consider the multi-sensor cases [6,17,25] because of the complex model structures of nonlinear systems. It is documented that the fusion problem of nonlinear systems has be solved by expanding first-order Taylor series of the state and measurement equations [14]. But for nonlinear systems, only using first-order Taylor series expansion will lead to a large estimation deviation and even make the filter diverge. So, this type of fusion filter is far restricted in the practical applications.

In order to get WMF for nonlinear systems, we need to know two things: the relationships among the noise statistics of different sensors and the relationships among the measurement functions of different sensors. Taking the additive Gaussian noise into account, we can deal with the noise statistics of linear relationships. If the measurement functions of different sensors have the linear relationships, we can achieve the globally optimal WMF via linear ways. However, if the measurement functions have the nonlinear relationships, it is a challenging problem how to get WMF to achieve the global optimality (functionally equivalent to the CMF), which will be solved in the present paper.

With the development of wireless sensor networks (WSNs), information fusion technology has got more and more attention [31]. Due to the enormous information of the large-scale WSNs, the CMF with a high-dimension measurement cannot satisfy the real-time processing requirement, thus WMF with a low-dimension measurement becomes an effective way to solve this kind of fusion problems [32]. Particularly, in networked control systems, the uncertainties such as random delays or packet losses [26,30,33] make the fusion problem more complex [29].

In the present paper, a universal weighted measurement fusion (WMF) algorithm is presented via using Taylor series expansion method for general nonlinear systems. Furthermore, using the Unscented Kalman Filter (UKF), a generalized nonlinear WMF-UKF algorithm is proposed. It is proven that the proposed WMF-UKF asymptotically approaches to the CMF-UKF with the increase of the order of Taylor series expansion, thus it has the asymptotic global optimality. The proposed algorithm can adjust the fusion accuracy by controlling the order of Taylor series expansion. Moreover, the WMF-UKF can obviously reduce the computational cost compared to the CMF-UKF, particularly in the large-scale WSNs.

2. Problem formulation

Consider a discrete-time nonlinear system with multiple sensors

$$\boldsymbol{x}(k+1) = \boldsymbol{f}(\boldsymbol{x}(k),k) + \boldsymbol{w}(k) \tag{1}$$

$$\boldsymbol{z}^{(j)}(k) = \boldsymbol{h}^{(j)}(\boldsymbol{x}(k),k) + \boldsymbol{v}^{(j)}(k), \quad j = 1,2,\ldots,L \tag{2}$$

where $\boldsymbol{f}(\cdot,\cdot) \in \Re^n$ is the nonlinear process function, $\boldsymbol{h}^{(j)}(\cdot,\cdot) \in \Re^{m_j}$ is the nonlinear measurement function, $\boldsymbol{x}(k) \in \Re^n$ is the state vector at time k, $\boldsymbol{z}^{(j)}(k) \in \Re^{m_j}$ is the measurement vector of the jth sensor, $\boldsymbol{w}(k) \in \Re^n$ is the system process noise, and $\boldsymbol{v}^{(j)}(k) \in \Re^{m_j}$ is the measurement noise of the jth sensor. It is also assumed that $\boldsymbol{w}(k)$ and $\boldsymbol{v}^{(j)}(k)$ are uncorrelated white noises with zero mean and variances $\boldsymbol{Q}_w$ and $\boldsymbol{R}^{(j)}$, that is

$$\mathrm{E}\left\{ \begin{bmatrix} \boldsymbol{w}(t) \\ \boldsymbol{v}^{(j)}(t) \end{bmatrix} \begin{bmatrix} \boldsymbol{w}^{\mathrm{T}}(k) & \boldsymbol{v}^{(l)\mathrm{T}}(k) \end{bmatrix} \right\} = \begin{bmatrix} \boldsymbol{Q}_w & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{R}^{(j)} \end{bmatrix} \delta_{tk}\delta_{jl} \tag{3}$$

where E denotes the mathematical expectation, the superscript T denotes the transpose, and δ_{tk} and δ_{jl} are the Kronecker delta functions, i.e., $\delta_{tt} = 1$ and $\delta_{tk} = 0$ $(t \neq k)$.

For the systems (1) and (2), the measurement equation of the centralized measurement fusion system (CMFS) is given as:

$$\boldsymbol{z}^{(0)}(k) = \boldsymbol{h}^{(0)}(\boldsymbol{x}(k),k) + \boldsymbol{v}^{(0)}(k) \tag{4}$$

where

$$\boldsymbol{z}^{(0)}(k) = [\boldsymbol{z}^{(1)\mathrm{T}}(k), \boldsymbol{z}^{(2)\mathrm{T}}(k), \ldots, \boldsymbol{z}^{(L)\mathrm{T}}(k)]^{\mathrm{T}} \tag{5}$$

$$\boldsymbol{h}^{(0)}(\boldsymbol{x}(k),k) = [\boldsymbol{h}^{(1)\mathrm{T}}(\boldsymbol{x}(k),k), \boldsymbol{h}^{(2)\mathrm{T}}(\boldsymbol{x}(k),k), \ldots, \boldsymbol{h}^{(L)\mathrm{T}}(\boldsymbol{x}(k),k)]^{\mathrm{T}} \tag{6}$$

$$\boldsymbol{v}^{(0)}(k) = [\boldsymbol{v}^{(1)\mathrm{T}}(k), \boldsymbol{v}^{(2)\mathrm{T}}(k), \ldots, \boldsymbol{v}^{(L)\mathrm{T}}(k)]^{\mathrm{T}} \tag{7}$$

and the covariance matrix of $\boldsymbol{v}^{(0)}(k)$ is given as

$$\boldsymbol{R}^{(0)} = \mathrm{diag}(\boldsymbol{R}^{(1)}, \boldsymbol{R}^{(2)}, \cdots, \boldsymbol{R}^{(L)}) \tag{8}$$

where the symbol diag($\bullet$) denotes a diagonal matrix.