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Performance evaluation of Cubature Kalman filter in a GPS/IMU tightly-coupled navigation system

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ABSTRACT

In a GPS/IMU tightly-coupled navigation system, the extended Kalman filter (EKF) is widely used to estimate the navigation states, due to its simpler implementation and lower computational load. However, the EKF is a first order approximation to the nonlinear system. When the nonlinearity of the system is high, the negligibility in higher order terms of the nonlinear system will degrade the estimation accuracy. In this paper, a nonlinear filtering method Cubature Kalman filter (CKF) is introduced and analysed through Taylor expansion showing the CKF's capability in capturing higher-order terms of nonlinear system. The analysis indicates that the CKF benefits only when implemented with nonlinear systems. To better show the merits of the CKF, a nonlinear attitude expression is introduced to the integrated navigation system. The performance comparison between the CKF and the EKF is examined based on the observability analysis. When the observability degree is low, the CKF performs better than the EKF in the integrated navigation systems. Otherwise, the CKF has similar performance as the EKF in the tightly-coupled navigation system. The CKF is also superior to the EKF in the GPS outage and large misalignment cases when the nonlinearity of the system is high.

1. Introduction

Since R.E. Kalman proposed his famous recursive method to solve discrete linear filtering problems in 1960 [1], the Kalman filter has been widely used in many applications. However, the KF's basic requirements in linearity and Gaussian-distributed noise are hard to meet in real world implementation. To make the KF applicable to nonlinear systems, the EKF, based on the first order Taylor term of nonlinear functions, is proposed. Although the EKF maintains the computationally efficient updated form of the KF, it suffers some drawbacks, one of which is the degradation in estimation accuracy, due to neglecting the higher-order terms of nonlinear system function [2].

Based on the deterministic sampling framework, the UKF and central difference Kalman filter (CDKF) use a series of sigma-points to propagate the states and covariance matrix [3,4]. The sigma points are deterministically calculated from the mean and square-root decomposition of the covariance matrix of the *a priori* random variable [5]. Both the UKF and CDKF belong to the sigma-point Kalman filter family. The main difference between them is the sigma-points generation methods [5,6]. The UKF generates sigma-points through unscented transformation, while the CDKF uses the Stirling's interpolation formula to produce sigma-points [5]. The UKF and CDKF can be treated as a second-order approximation to a nonlinear system. So in theory they have higher estimation accuracy than the EKF. The UKF and CDKF have been applied in the GPS/IMU integrated navigation system and perform better than the EKF as introduced in [5,7–10].

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The CKF is a recently developed nonlinear filtering method based on the spherical-radial Cubature rule, which is developed to compute integrals like nonlinear function times Gaussian density [11–13]. The CKF can be treated as a second-order approximation to a nonlinear system. The higher-order CKF is also proposed as a more accurate approximation to a nonlinear system [14–16]. Unlike the UKF using $2n+1$ unscented points to propagate the state and covariance matrix, the CKF propagate the state and covariance matrix with $2n$ Cubature points, due to which the CKF has a relatively lower computational load than the UKF when the same matrix decomposition methods like singular value decomposition or cholesky method are applied to the UKF and CKF. Although the Cubature points are applied, the CKF still belongs to sigma-point Kalman filter family. The CKF shows better performance than the UKF in stability, especially when the dimension of the system is higher than 3 as suggested in [11–13]. As indicated in the UKF, the choice of κ must satisfy $n+\kappa=3$. If the dimensionality or number of nonlinear equations is higher than three, κ is negative, which may render the covariance matrix negative definite. The CKF will not suffer from such a problem since all the weights in the CKF are positive, guaranteeing the positive definiteness of the covariance matrix of the filtering process. The CKF and its extension have been implemented in many applications. The Cubature information filter is proposed and applied in decentralized fusion in [17]. The CKF is applied to a GNSS/INS tightly-coupled navigation system and the navigation estimation is enhanced as shown in [18,19]. An interactive multi model (IMM)-CKF method is applied to estimate the mobile station's location in [20]. In [21], a square-root adaptive CKF is applied in the spacecraft attitude estimation. In [22,23], a Cubature $H\infty$ filter and its square-root version are proposed and verified in a continuous stirred tank reactor and a permanent magnet synchronous motor as examples. The CKF can also be applied to the SLAM (Simultaneous Localisation And Mapping) problem as reported in [24].

In this paper, the performance of the CKF in GPS/IMU tightly-coupled navigation systems will be researched in theory and experimentally. The EKF is implemented as a comparison to check the CKF performance.

2. Filtering algorithms

This section briefly introduces the CKF and EKF algorithms. Considering a discrete nonlinear system as

$$\begin{aligned}\mathbf{x}_k &= \mathbf{f}(\mathbf{x}_{k-1})+\mathbf{w}_{k-1}\\ \mathbf{z}_k &= \mathbf{h}(\mathbf{x}_k)+\mathbf{v}_k\end{aligned} \tag{1}$$

where $\mathbf{x}_k \in \Re^{\mathbf{n_x}}$ is system's state vector at time epoch k, $\mathbf{z}_k \in \Re^{\mathbf{n_z}}$ is measurement, $\mathbf{w}_{k-1} \in \Re^{\mathbf{n_w}}$ and $\mathbf{v}_k \in \Re^{\mathbf{n_v}}$ represent independent process and measurement Gaussian noise sequences assumed to be independent, white and with covariance $\mathbf{Q}_k$ and $\mathbf{R}_k$ in respect.

2.1. Extended Kalman filter

The EKF solves nonlinear problems by approximating nonlinear functions using the first order term of Taylor expansion. The Jacobian matrices calculated from the nonlinear state transition function and measurement function are implemented in the EKF as state transition and measurement matrices. The EKF algorithm is summarised as

(i) Time Update

$$\begin{aligned}\mathbf{x}_{k|k-1} &= \mathbf{\Phi}_k\mathbf{x}_{k-1|k-1}\\ \mathbf{P}_{k|k-1} &= \mathbf{\Phi}_k\mathbf{P}_{k-1|k-1}\mathbf{\Phi}_k^T+\mathbf{Q}_k\end{aligned} \tag{2}$$

(ii) Measurement Update

$$\begin{aligned}\mathbf{K}_k &= \mathbf{P}_{k|k-1}\mathbf{H}_k^T(\mathbf{H}_k\mathbf{P}_{k|k-1}\mathbf{H}_k^T+\mathbf{R}_k)^{-1}\\ \mathbf{x}_{k|k} &= \mathbf{x}_{k|k-1}+\mathbf{K}_k(\mathbf{z}-\mathbf{H}_k\mathbf{x}_{k|k-1})\\ \mathbf{P}_{k|k} &= \mathbf{P}_{k|k-1}-\mathbf{K}_k\mathbf{H}_k\mathbf{P}_{k|k-1}\end{aligned} \tag{3}$$

where $\mathbf{\Phi}_k$ is the state transition matrix, which is the Jacobian matrix of the nonlinear function $\mathbf{f}(\cdot)$, and $\mathbf{H}_k$ is the measurement matrix, which is the Jacobian matrix of the nonlinear function $\mathbf{h}(\cdot)$. $\mathbf{K}_k$ is the Kalman gain.

2.2. Cubature Kalman filter

The CKF uses a series of Cubature points to propagate the *a priori* and *a posteriori* statistical characteristics. The core of the CKF is the Cubature transformation based on the spherical-radial rule [11]. The CKF algorithm is summarized as follows:

(i) Time Update

$$\begin{aligned}\mathbf{S}_{k-1|k-1} &= SVD(\mathbf{P}_{k-1|k-1})\\ \chi_{k-1|k-1} &= \mathbf{S}_{k-1|k-1}\xi+\mathbf{x}_{k-1|k-1}\\ \chi^*_{k|k-1} &= \mathbf{f}(\chi_{k-1|k-1})\\ \mathbf{x}_{k|k-1} &= \frac{1}{m}\sum_{i=1}^{m}\chi^*_{i,k|k-1}\\ \mathbf{P}_{k|k-1} &= \frac{1}{m}\sum_{i=1}^{m}\chi^*_{i,k|k-1}\chi^{*T}_{i,k|k-1}-\mathbf{x}_{k|k-1}\mathbf{x}^T_{k|k-1}+\mathbf{Q}_k\end{aligned} \tag{4}$$

(ii) Measurement Update

$$\begin{aligned}\mathbf{S}_{k|k-1} &= SVD(\mathbf{P}_{k|k-1})\\ \chi_{k|k-1} &= \mathbf{S}_{k|k-1}\xi+\mathbf{x}_{k|k-1}\\ \mathbf{Z}_{k|k-1} &= \mathbf{h}(\chi_{k|k-1})\\ \mathbf{z}_{k|k-1} &= \frac{1}{m}\sum_{i=1}^{m}\mathbf{Z}_{i,k|k-1}\\ \mathbf{P}_{\mathbf{zz},k|k-1} &= \frac{1}{m}\sum_{i=1}^{m}\mathbf{Z}_{i,k|k-1}\mathbf{Z}^T_{i,k|k-1}-\mathbf{z}_{k|k-1}\mathbf{z}^T_{k|k-1}+\mathbf{R}_k\\ \mathbf{P}_{\mathbf{xz},k|k-1} &= \frac{1}{m}\sum_{i=1}^{m}\chi_{i,k|k-1}\mathbf{Z}^T_{i,k|k-1}-\mathbf{x}_{k|k-1}\mathbf{z}^T_{k|k-1}\\ \mathbf{K}_k &= \mathbf{P}_{\mathbf{xz},k|k-1}\mathbf{P}^{-1}_{\mathbf{zz},k|k-1}\\ \mathbf{x}_{k|k} &= \mathbf{x}_{k|k-1}+\mathbf{K}_k(\mathbf{z}_k-\mathbf{z}_{k|k-1})\\ \mathbf{P}_{k|k} &= \mathbf{P}_{k|k-1}-\mathbf{K}_k\mathbf{P}_{\mathbf{zz},k|k-1}\mathbf{K}_k^T\end{aligned} \tag{5}$$

where SVD represents the matrix singular value decomposition method, $\mathbf{S}$ is the square-root of the covariance matrix $\mathbf{P}$, $m=2n$, $\xi=\sqrt{m/2}[1]_i$, χ_i is the Cubature point generated from states and $\mathbf{Z}_i$ represents the Cubature point generated from measurements.

The CKF uses $2n$ Cubature points to propagate state and covariance matrix. The calculation of Jacobian matrix is avoided.