Aerospace Science and Technology 51 (2016) 192–202

Contents lists available at ScienceDirect

Aerospace Science and Technology

www.elsevier.com/locate/aescte

A polynomial chaos based square-root Kalman filter for Mars entry navigation

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ARTICLE INFO

Article history:
Received 13 June 2013
Received in revised form 26 November 2015
Accepted 6 February 2016
Available online 10 February 2016

Keywords:
Mars entry
Autonomous navigation
Polynomial chaos
Square-root Kalman filter

ABSTRACT

In order to compute the sequential state estimation of Mars entry dynamic system from noisy observations, a deterministic square-root Kalman filter is developed with the implementation of polynomial chaos expansion. The filter allows for the nonlinearity of dynamic system and observation model without the need for assumption about the Gaussian distribution of state. In the algorithm, a minimum variance based data assimilation scheme is developed. The resulting Kalman type updates of state's mean and deviations are performed separately and the square-root formulation of coefficients of polynomial chaos can be computed directly from the polynomial chaos expansion of forecasted states and observations. Two autonomous navigation scenarios based on the Mars Network are considered to quantify the benefits of polynomial chaos based square-root Kalman filter over the other usual nonlinear filters. Additionally, the contrastive analysis of propagations with non-Gaussian states is conducted. Simulation results show that a more accurate estimation and faster convergence can be achieved from the proposed nonlinear filter. Finally, the accuracy and time efficiency of the polynomial chaos based square-root Kalman filter are further analyzed and the optimal order of polynomial chaos is recommended.

1. Introduction

The focus of Mars exploration mission design is being shifted from mission safety to scientific goals. Future Mars missions may thus need the capability of precise landing at predefined location of great scientific interests [1]. NASA has also emphasized the need for fundamental researches on pinpoint landing of large robotic and manned spacecrafts on Mars in the future. Autonomous navigation technologies in the Mars entry phase will be crucial to meet accuracy requirements for future missions. Up to date, NASA has successfully landed seven robotic systems on the surface of Mars, and all of them were navigated by the dead reckoning system developed for Viking missions. For the improvement of navigation performance, the navigation scheme based on the Mars Network has been proposed [2,3]. Unlike the atmospheric entry on the earth, the initial state of entry vehicle can not be accurately determined due to limited navigation measurements. This may lead to relative large and non-Gaussian initial errors. Furthermore, the duration of entry phase is very short. Therefore, real time state estimation with high accuracy and fast convergence will be the key to Mars entry navigation.

Multiple sub-optimal filters for state estimation performed in Bayesian framework have been investigated. The most widely used approach is the extended Kalman filter (EKF) [4,5]. For nonlinear dynamic systems, Taylor based linearization method is employed so that state propagation and assimilation can be performed with the locally linear system. However, analytic dynamic and observation models have to be provided, which may not be satisfied in actual application. Furthermore, the introduced truncation errors may slow down the convergence and decrease the accuracy.

The estimation error may be reduced if the state propagation and assimilation can be performed directly through the nonlinear system. The Monte Carlo based Kalman type filters aiming to solve this issue have been of interest to investigators in recent years. The ensemble Kalman filter (EnKF), which was first proposed by Evensen [6], has been adapted in a wide field of application. In the algorithm, a set of ensemble elements is generated using Monte Carlo sampling for the uncertain states, modeling errors, and ob-

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http://dx.doi.org/10.1016/j.ast.2016.02.009
1270-9638/

servation noise. Then ensemble elements are propagated through nonlinear dynamic model and are analyzed by Kalman formulation. The state covariance is approximated by ensemble covariance, so that the manipulation and storage of state covariance may be avoided. EnKF is more practical for large state space system for which the covariance matrix may be too large. Several variations of EnKF, in particular ensemble square-root filter (EnSRF) [7–9], have been developed to gain efficiency.

Particle filters (PF) are also ensemble based assimilation methodologies dealing with the nonlinear dynamic system and non-Gaussian uncertainties, and have been applied in the GPS/INS navigation and target tracking [10,11]. But they are not practical for high dimensional systems because of inefficient sampling [12]. Similar approaches include the unscented Kalman filter (UKF) [13–15] in which the Unscented Transformation (UT) is introduced. In the algorithm, a series of sigma points is employed instead of the Monte Carlo sampling to capture the mean and error covariance. The detailed comparison of EKF, PF, and UKF can be found in [16]. Recently, the cubature Kalman filter (CKF), which is another numerical approximation of sequential Bayesian filter, has been proposed and widely used [17–20]. The CKF is based on the spherical-radial cubature rule computing the multivariate moment integrals numerically. It has been proved that the third-degree cubature rule is a special form of UT with better numerical stability [17]. Therefore, compared with UKF, only a limited improvement of accuracy may be achieved [20].

On the other hand, if the statistical properties of model error and observation noise are unknown or can not be accurately characterized, robust H-infinity filters may be better choices [21–23]. Compared with Kalman-type filters, H-infinity filters provide the robustness by limiting the worst-case estimation errors. However, if the noise statistics are known, H-infinity filters may not guarantee a better accuracy [21].

The polynomial chaos (PC), which was first theoretically presented by Wiener in 1938 [24], has been successfully evolved into the entire Askey scheme of orthogonal polynomials [25]. The polynomial chaos framework has been proven to be more computationally efficient than Monte Carlo approach for propagation uncertainties in stochastic systems [26,27] and has already been used in the state and parameter estimation. Pence et al. [28] combined the polynomial chaos theory with the maximum likelihood estimation and proposed a gradient based parameter update law. Li and Xiu [29] developed an efficient EnKF implementation combining with the polynomial chaos expansion, in which they sampled the polynomial chaos expression of the state in order to update the coefficient of polynomial chaos with high accuracy. Pajonk [30] proposed a polynomial chaos based non-Gaussian Bayesian estimator for state estimates of dynamic models, which did not need the assumptions about Gaussian distribution of states. Pajonk [31] also developed square-root approach to update the coefficients of polynomial chaos. Blanchard [32] presented the polynomial chaos based Kalman filter for parameter estimation of mechanical systems, in which the error covariance matrix was computed by polynomial chaos expansion, and the Kalman based update was utilized to estimate the polynomial chaos representation of uncertain states.

Combining PC theory with the Kalman type updating scheme is a way of gaining efficiency. However, in most of previous PC based filters, state estimations were performed using linear observation model, which is not suitable for Mars entry navigation scheme [29,32]. Meanwhile, in the search of [30,32], all coefficients of polynomial chaos are updated with the same Kalman based gain matrix by sampling the uncertainty of observations. Therefore, the efficiency is reduced and more sampling error may be introduced. Pajonk [31] considered the nonlinear observation model, and updated the coefficients of polynomial chaos separately by square-root approach. However, expect for the zero order coefficient of polynomial chaos, the other coefficients were updated based on a linear measurement operator which led to a linear transformation between posterior and prior *covariance of square root*.

This paper extends the polynomial chaos theory to the nonlinear state estimation problem based on the square-root implementation and proposes the polynomial chaos based square-root Kalman filter (PCSKF). The dynamic system with accurate dynamic model and known statistical properties of observation noise is emphasized. In the algorithm, the point-collocation polynomial chaos expansion is employed in the propagation phase allowing for the nonlinear dynamic model. In the updating phase, the covariance matrices are computed based on the polynomial chaos expansion of forecasted states and observations. Then the updating matrices are calculated through the square-root approach. The PCSKF takes account of the nonlinearity of dynamic model and observation model, and does not need the assumption about Gaussian distributed states.

The organization of the paper is as follows. Following a brief review of the Mars Network based Mars entry navigation in Section 2, and the basic theory of polynomial chaos expansion for random process is presented in Section 3. In Section 4, the algorithm of polynomial chaos based square-root Kalman filter is developed, which is the main part of the paper. Then, in Section 5, the performance and benefits of PCSKF for Mars entry navigation are quantified via a series of comparative analysis. Section 6, finally, provides concluding remarks of this research.

2. Mars Network based Mars entry navigation scheme

Recent researches indicated that high frequency radio signals could penetrate the ionizing plasma sheath around the entry vehicle when it entered the Martian atmosphere [33]. This investigation paved the way for the future autonomous Mars entry navigation based on the Mars Network, which is illustrated in Fig. 1. With high frequency, such as ultrahigh frequency transceivers which are mounted on the Mars orbiters or ground based beacons, spacecraft-to-spacecraft or spacecraft-to-beacon radial range and velocity can be provided as observed quantities for the navigation system. The dynamic model and the observation model of the Mars entry navigation based on Mars Network are introduced above all.

For the sake of simplicity, the common assumptions for Mars entry dynamics are defined as follows:

1) The planet of Mars has a spherical shape,
2) the atmosphere is steady,
3) the atmospheric density has an exponential behavior,
4) the direction of aerodynamic lift force is controllable.

Based on these assumptions, a 3-degree-of-freedom dynamic model of Mars entry is developed taking account of the Corioli's effect.

$$
\begin{aligned}
\dot{r} &= V \sin\gamma \\
\dot{\theta} &= V \cos\gamma \cos\psi/(r\cos\phi) \\
\dot{\phi} &= V \cos\gamma \sin\psi/r \\
\dot{V} &= -D - g\sin\gamma \\
\dot{\gamma} &= [L\cos\sigma - (g - V^2/r)\cos\gamma]/V \\
&\quad + 2\omega(\tan\gamma \sin\psi \cos\phi - \sin\phi) \\
\dot{\psi} &= -(L\sin\sigma + V^2\cos^2\gamma \cos\psi \tan\phi/r)/(V\cos\gamma) \\
&\quad + 2\omega\cos\psi\cos\phi
\end{aligned}
\tag{1}
$$

In the dynamic model, the six dimensional states include r (radial position from the center of the planet to the entry vehicle's