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Analysis of a robust Kalman filter in loosely coupled GPS/INS navigation system

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ABSTRACT

GPS/INS integrated system is very subject to uncertainties due to exogenous disturbances, device damage, and inaccurate sensor noise statistics. Conventional Kalman filer has no robustness to address system uncertainties which may corrupt filter performance and even cause filter divergence. Based on the INS error dynamic equation, a robust Kalman filter is analyzed and applied in loosely coupled GPS/INS integration system. The norm bounded robust Kalman filter, with recursive form by solving two Riccati equations, guarantees a estimation variance bound for all the admissible uncertainties, and can evolve into the conventional Kalman filter if no uncertainties are considered. This paper will analyze the suitable case for the robust Kalman filter in GPS/INS system, the filter characteristics including parameter setting, parameter meaning, and filter convergence condition are discussed simutaneously. The robust filter performance will be compared with conventional Kalman filter through simulation results.

1. Introduction

In the past decades, benifiting from overcoming each others' limitations and performance improvement, global positioning system (GPS) and inertial navigation system (INS) integrated GPS/INS system had become a standard approach in modern navigation area. The three main coupled ways: loosely coupled integration, tight coupled integration, and ultra-tight [1] coupled integration, are all designed based on the INS error dynamic model, i.e. using the GPS information to calibrate INS accumulation error with navigation filtering technique. Kalman filter has tacitly become a standard data fusion approach in GPS/INS integration system. However, for real time application, a conventional Kalman filter is not enough. Traditional filter modification often lie in two aspects: Using extended Kalman filter (EKF), unscented Kalman filter (UKF), sigma-point Kalman filter (SPFK) [9,18] to cope with model nonlinearity, and using adaptive Kalman filter [10,4] to address the blunder error or such abnormal factors.

Nevertheless, based on our understanding, there is rare research in uncertainty area for GPS/INS integration system. Motivated by the duality of optimal control and filtering, we analyze and apply a robust Kalman filter in loosely coupled GPS/INS integration system. Fundamentally, for an uncertainty robust problem, there are two main adopts: one is using a coupled Riccati equations to guarantee the pre-set uncertain bound, the other is using the real bounded lemma to convert uncertainty into a series of linear matrix inequality(LMI) constrains, and solved it as a convex optimal problem. Although the LMI approach is more efficient and less conservatism, they are often designed for a linear time invariant (LTI) system. However, the INS error dynamic model is a time-varying system whose parameters are determined by real time position, velocity, and attitude i.e. the transition matrix parameters

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depend on the real time navigation output. The Riccati approach apparently has a resemble structure as the conventional Kalman filter, further more, it is easily used in time-domain estimation process. This kind of robust Kalman filter, which is also called minimum-variance or variance constrain filter, was firstly discussed with norm-bounded uncertainty in [12,14], following various of extensions [15,17,11] with extra constrains. This robust filter depicts uncertainty as a time-varying but norm bounded parameter, then the estimation covariance will guarantee a upper bound for all admissible uncertainty. The $H\infty$ theory, as an alternative approach for state estimation when robustness is considered, shows better performance in the absence of accurate noise statistics [13]. A standard $H\infty$ filter for GPS/INS model is discussed in [19], while none system uncertainty had been concerned. And the main contribution of this paper lies in that we analyzed a norm bounded Kalman filter in GPS/INS system, explained the robust filter parameters meaning in GPS/INS system, and used $H\infty$ theory to verify robust Kalman filter convergent condition. We derived how the robust filter reduced to conventional Kalman filter when no uncertainty is concerned, and eventually discussed the suitable case for a robust Kalman filter in GPS/INS system.

The rest parts of this paper are organized as follows. Section 2 preliminarily introduced the robust Kalman filter design procedure, and derived that the robust filter would reduce to conventional Kalman filter without concerning uncertainty. In Section 3, we unitized $H\infty$ theory to analyze the robust Kalman filter performance and give an explanation of the robust filter parameters setting. After that, Section 4 fundamentally reviewed the INS error model and conventional navigation filter design. Finally, simulation results will be given to test the robust filter with contrast to convention Kalman filter in Section 5.

2. Robust Kalman filter design

This section firstly introduces an uncertain system with norm bounded depiction, then we derive the robust Kalman filter formulation. At last, we discuss the relationship between conventional Kalman filter and the robust Kalman filter.

2.1. Norm bounded system

Consider a discrete time varying uncertain system:

$$\begin{aligned} x_{k+1} &= (A_k + H_{1,k}F_kE_k)x_k + w_k \\ y_k &= (C_k + H_{2,k}F_kE_k)x_k + v_k \end{aligned} \tag{1}$$

where $x_k \in R^n$ is the state, $y_k \in R^p$ is the measurement, $A_k,\ C_k,\ H_{1,k},\ H_{2,k},\ E_k$ are known real time-varying matrices with appropriate dimensions. F_k is the time-varying norm bounded uncertainty matrix, $\forall k$, fulfilling $F_kF_k^T \leqslant I$, w_k and v_k are Gaussian white noise sequence for process and measurement noise respectively, with the expectation and covariance:

$$\begin{aligned} &E[w_k] = 0 \quad E[v_k] = 0 \\ &E[w_kw_m^T] = Q\delta_{k,m} \quad E[v_kv_m^T] = R\delta_{k,m} \end{aligned} \tag{2}$$

where $Q \in R^{n\times n},\ R \in R^{p\times p},\ \delta_{k,m}$ is the Kronecker delta function which equals to unity when $k = m$, and 0 for the rest. It is worth mentioning that there is no dimension requirement for the uncertainty matrix F_k, since it can be absorbed in H and E, in other words, so long as its column number coincides with H, and row number coincides with E.

2.2. Covariance bounded constrain analysis

A filter for dynamic discrete system can be written

$$\hat{x}_{k+1} = \hat{A}_k\hat{x}_k + \hat{K}_k(y_k - C_k\hat{x}_k) \tag{3}$$

The objective of the robust filter problem is to determine the parameters $\hat{A}_k$ and $\hat{K}_k$ in order to guarantee the estimation error is bounded for all the admissible uncertainty F_k, i.e. there is a sequence of positive definite symmetric matrices Θ_k such that $\forall k$:

$$E[(x_k - \hat{x}_k)(x_k - \hat{x}_k)^T] \leqslant \Theta_k \tag{4}$$

and Θ_k can be determined in advance according to the uncertainty scalar matrix H and E. Here we introduce an augment system which is widely used in H_2 filter and control problems. Define a state vector:

$$\tilde{x}_k = [x_k \quad \hat{x}_k]^T \tag{5}$$

so the augment system dynamic equation can be expressed as:

$$\tilde{x}_{k+1} = (\tilde{A}_k + \tilde{H}_kF_k\tilde{E}_k)\tilde{x}_k + \tilde{B}_kd_k \tag{6}$$

where

$$\begin{aligned} &\tilde{A}_k = \begin{bmatrix} A_k & 0 \\ \hat{K}_kC_k & \hat{A}_k - \hat{K}_kC_k \end{bmatrix} \quad \tilde{H}_k = \begin{bmatrix} H_{1,k} \\ \hat{K}_kH_{2,k} \end{bmatrix} \\ &\tilde{E}_k = \begin{bmatrix} E_k \\ 0 \end{bmatrix}^T \quad \tilde{B}_k = \begin{bmatrix} I & 0 \\ 0 & \hat{K}_k \end{bmatrix} \quad d_k = \begin{bmatrix} w_k \\ v_k \end{bmatrix} \end{aligned} \tag{7}$$

Denote the covariance of $\tilde{x}_k$ as $\tilde{X}_k = E[\tilde{x}_k\tilde{x}_k^T]$, we straightforward get that:

$$\tilde{X}_{k+1} = (\tilde{A}_k + \tilde{H}_kF_k\tilde{E}_k)\tilde{X}_k(\tilde{A}_k + \tilde{H}_kF_k\tilde{E}_k)^T + \tilde{B}_k\Phi\tilde{B}_k^T \tag{8}$$

where $\Phi = \begin{bmatrix} Q & 0 \\ 0 & R \end{bmatrix}$. It is impossible to calculate exact estimation covariance because we confront a uncertainty matrix F. An alternative approach is to build a fixed bound which can contain all the admissible uncertainty. A candidate upper bound is given by the lemmas below.

Lemma 1 [3]. *Let M be a symmetric matrix, $A,\ H,\ E$, are real matrices, X be symmetric positive-definite matrix, and a matrix F satisfy $FF^T \leqslant I$. We obtain:*

$$(A + HFE)X(A + HFE)^T - M \leqslant 0 \tag{9}$$

if and only if there is a constant $\alpha > 0$ such that: