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# Distributed Adaptive High-Gain Extended Kalman Filtering for Nonlinear systems

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**Abstract:** In this work, we propose a distributed adaptive high-gain extended Kalman filtering approach for nonlinear systems. Specifically, we consider a class of nonlinear systems that are composed of several subsystems interacting with each other via their states. In the proposed approach, an adaptive high-gain extended Kalman filter is designed for each subsystem. The distributed Kalman filters communicate with each other to exchange subsystem state estimates. First, an implementation strategy which specifies how the distributed filters should communicate is designed. Second, the detailed design of the subsystem filter is described. Subsequently, the stability of the proposed distributed state estimation is analyzed. Finally, the effectiveness and applicability of the proposed design are illustrated via the application to a chemical process example.



*Keywords:* Distributed state estimation; Extended Kalman filter; Nonlinear systems; High gain observer.

## 1. INTRODUCTION

Distributed state estimation of large-scale systems composed of coupled subsystems has attracted significant attention in process control (Farina et al. (2011); Zhang and Liu (2013)) as well as in many other engineering control problems (Khan and Moura (2008); Stanković et al. (2009)). In the literature, distributed state estimation has been studied primarily in two frameworks: distributed Kalman filtering (Mahmoud and Khalid (2013)) and distributed moving horizon state estimation (Farina et al. (2011); Zhang and Liu (2013)).

Kalman filters are widely used to obtain system state estimates in many applications ranging from industrial processes (Chetouani et al. (2002)) to aerospace navigation systems (Madrid and Bierman (1978)). For applications involving large-scale systems, centralized state estimation algorithms are in general not favorable due to organizational difficulties, high computational complexity and poor fault tolerance. In order to address these considerations, distributed Kalman filtering has been studied extensively in the past decade (Mahmoud and Khalid (2013)). A large portion of the existing results on distributed Kalman filtering (DKF) were developed for sensor networks (Stanković et al. (2009)). Within process control, a method to decompose large-scale processes for distributed Kalman filtering and distributed control was presented in Vadigepalli and Doyle III (2003). Most of the above results were obtained in the context of linear systems. When nonlinear systems are present, extended Kalman filters (EKF) are typically used in the design of distributed state estimation algorithms (Mutambara (1998)). As in the centralized EKF, the global stability of the error dynamics of the distributed EKF is difficult to establish.

In recent years, moving horizon state estimation (MHE) has been adopted in distributed state estimation. One of the advantages of MHE is its ability to account for state constraints which have been shown leading to improved estimates (Rao et al. (2001)). In Farina et al. (2010), a distributed MHE algorithm was developed for linear systems which was extended to nonlinear systems in Farina et al. (2012). In Farina et al. (2011), distributed MHE schemes based on subsystem models were developed for both linear and nonlinear systems. In Zhang and Liu (2013), an observer-enhanced distributed MHE design with potentially tunable convergence rate was introduced. A method for handling communication delays in distributed MHE was also developed in Zhang and Liu (2014). While distributed MHE algorithms are able to handle system nonlinearities explicitly, they are typically much more computational demanding than Kalman filters.

Motivated by the above considerations, in this work, we propose a distributed adaptive high-gain extended Kalman filtering approach for nonlinear systems. The proposed approach is able to achieve global stability of the error dynamics and is computationally efficient. Specifically, we consider a class of continuous-time nonlinear systems that are composed of several subsystems interacting with each other via their states. In the proposed approach, an adaptive high-gain extended Kalman filter (AHG-EKF) is designed for each subsystem. The distributed Kalman filters communicate with each other to exchange subsystem state estimates. First, the implementation strategy of the proposed distributed state estimation is discussed. It specifies how the distributed filters communicate and what information should be exchanged. Second, the detailed design of the subsystem filter is described. Subsequently,

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the stability of the distributed state estimation design is analyzed. Finally, the effectiveness and applicability of the proposed design are illustrated via the application to a chemical process example.

## 2. PRELIMINARIES

### *2.1 System description and observability assumption*

We consider nonlinear systems composed of $p$ interconnected subsystems with the dynamics of subsystem $i$ described as follows:

$$\begin{aligned}\dot{\hat{x}}_i(t) &= f_i(\hat{x}_i(t)) + \tilde{f}_i(\hat{X}_i(t)),\\ y_i &= h_i(\hat{x}_i)\end{aligned} \tag{1}$$

where $i \in \mathbb{I}$ with $\mathbb{I} = \{1, \ldots, p\}$, $\hat{x}_i \in \mathbb{R}^{n_{x_i}}$ is the state vector of subsystem $i$, the vector function $\tilde{f}_i$ represents the interaction term between subsystem $i$ and other subsystems with $\hat{X}_i$ composed of subsystem states involved in characterizing the interaction, and $y_i \in \mathbb{R}$ denotes the measured output of subsystem $i$. Note that to simplify the analysis without loss of generality, we assume that each subsystem has only one measured output.

The entire system state vector $\hat{x}$ and measured output vector $y$ are defined as $\hat{x} = [\hat{x}_1^T \cdots \hat{x}_i^T \cdots \hat{x}_p^T]^T \in \mathbb{R}^{n_x}$, $y = [y_1 \cdots y_i \cdots y_p]^T \in \mathbb{R}^p$. The dynamics of the entire system can be described by the following state-space model:

$$\begin{aligned}\dot{\hat{x}}(t) &= f(\hat{x}(t))\\ y(t) &= h(\hat{x}(t))\end{aligned} \tag{2}$$

where $f$ is an appropriate composition of $f_i$ and $\tilde{f}_i$, and $h$ is a composition of $h_i$ with $i \in \mathbb{I}$. It is assumed that the subsystem state $\hat{x}_i$ satisfies $\hat{x}_i \in \mathbb{X}_i$ for all $i \in \mathbb{I}$, and the measurements of the outputs of the subsystems are available continuously. It is also assumed that subsystem $i$, $i \in \mathbb{I}$, is observable in $\mathbb{X}_i$ for all $\hat{x}_j \in \mathbb{X}_j$ ($j \neq i$, $j \in \mathbb{I}$) in the sense that $\mathrm{rank}(O_i(\hat{x}_i)) = n_{x_i}$ where $O_i(\hat{x}_i) = \dfrac{d\Phi_i(\hat{x}_i)}{d\hat{x}_i}$ (Isidori (1995)) with

$$\Phi_i(\hat{x}_i) = \begin{bmatrix} h_i(\hat{x}_i) \\ L_{f_i+\tilde{f}_i} h_i(\hat{x}_i) \\ \vdots \\ L_{f_i+\tilde{f}_i}^{n_{x_i}-1} h_i(\hat{x}_i) \end{bmatrix} \tag{3}$$

Note that in (3), the symbol $L_{f_i+\tilde{f}_i} h_i$ denotes the Lie derivative of function $h_i$ with respect to $f_i + \tilde{f}_i$, defined as $L_{f_i+\tilde{f}_i} h_i = \dfrac{\partial h_i}{\partial \hat{x}_i}(f_i + \tilde{f}_i)$ while $L^r_{f_i+\tilde{f}_i} h_i$ denotes $r$-th order Lie derivative, defined as $L^r_{f_i+\tilde{f}_i} h_i = L_{f_i+\tilde{f}_i} L^{r-1}_{f_i+\tilde{f}_i} h_i$.

It is further assumed that the entire system of Eq. (2) is observable. This means that the interactions between subsystems do not affect the collective observability of the subsystems.

### *2.2 Change of coordinates*

From the observability assumptions (i.e. $\mathrm{rank}(O_i(\hat{x}_i)) = n_{x_i}$, $i \in \mathbb{I}$) and the implicit function theorem, it can be obtained that $\Phi_i(\hat{x}_i)$, $i \in \mathbb{I}$, is invertible for all $\hat{x}_i \in \mathbb{X}_i$, $i \in \mathbb{I}$. The function $\Phi_i(\hat{x}_i)$, $i \in \mathbb{I}$, also defines a set of

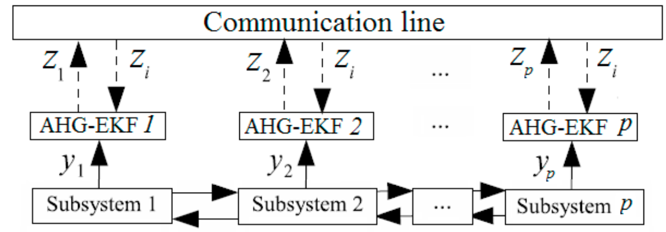


Fig. 1. Proposed distributed state estimation design.

coordinates under which system (1) can be transformed to a canonical form. Specifically, the new set of coordinates is defined as follows:

$$x_i = \begin{bmatrix} x_{i1} & x_{i2} & \ldots & x_{i,n_{x_i}} \end{bmatrix}^T = \Phi_i(\hat{x}_i) \tag{4}$$

In the new set of coordinates, subsystem (1) can be written in the following form:

$$\begin{aligned}\dot{x}_i &= A_i x_i(t) + b_i(X_i)\\ y_i &= C_i x_i(t)\end{aligned} \tag{5}$$

where $C_i = [1, 0, \ldots, 0]$ and

$$A_i = \begin{bmatrix} 0 & 1 & 0 & \ldots & 0 \\ & & & \ddots & \vdots \\ & 0 & 1 & & \\ \vdots & & \ddots & \ddots & 0 \\ & & & 0 & 1 \\ 0 & & \ldots & & 0 \end{bmatrix}, b_i(X_i) = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ L^{n_{x_i}-1}_{f_i+\tilde{f}_i} h_i(\Phi_i^{-1}(x_i(t))) \end{bmatrix} \tag{6}$$

where $X_i$ denotes the composition of all the subsystem states characterizing the interaction between subsystem $i$ and other subsystems in new coordinates.

In the remainder of this paper, we will discuss the proposed distributed state estimation design and its stability properties based on system (5).

## 3. PROPOSED DISTRIBUTED STATE ESTIMATION

In this section, we introduce the proposed distributed state estimation design. A schematic of the proposed design is shown in Figure 1. In the proposed design, an AHG-EKF is designed for each subsystem. The different AHG-EKFs communicate with each other to exchange information. The filter of a subsystem estimates the subsystem state based on the subsystem output measurements and information exchanged with other filters. In the following subsections, we will first discuss the implementation strategy and then describe the design of each subsystem AHG-EKF.

### *3.1 Implementation strategy*

As mentioned earlier, in this work, we assume that the output measurements are available continuously. To simplify the filter design and stability analysis without loss of generality, we further assume that the distributed filters can communicate and exchange information continuously.

At the initial time instant (i.e., $t = 0$), each filter needs to be initialized. Specifically, in the initialization, filter $i$ ($i \in \mathbb{I}$) is initialized with initial subsystem state guesses of all the subsystems (i.e., $z_j(0)$ with $j \in \mathbb{I}$), the actual subsystem output measurement (i.e., $y_i(0)$), and the initial value of its adaptive gain (i.e., $\theta_i(0) = 1$).