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Feedback control and Kalman Filtering of nonlinear wave dynamics

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Abstract: The paper proposes a pointwise control method for the 1D nonlinear wave equation and a filtering approach for estimating the dynamics of such a system from measurements provided by a small number of sensors. It is shown that the numerical solution of the associated partial differential equation results into a set of nonlinear ordinary differential equations. With the application of a diffeomorphism that is based on differential flatness theory it is shown that an equivalent description of the system in the linear canonical (Brunovsky) form is obtained. This transformation enables to obtain estimates about the state vector of the system through the application of the standard Kalman Filter recursion. For the local subsystems, into which the nonlinear wave equation is decomposed, it becomes possible to apply pointwise state estimation-based feedback control. The efficiency of the proposed filtering and control approach for nonlinear systems described by 1D partial differential equations of the wave type (e.g. sine-Gordon PDE) is confirmed through simulation experiments.

Keywords: differential flatness theory, nonlinear Kalman Filter, nonlinear wave equations.

1. INTRODUCTION

Nonlinear wave-type differential equations is met in communication systems (transmission lines, optical fibers and electromagnetic waves propagation), in electronics (Josephson junctions), in fluid flow models, in structural engineering (dynamic analysis of buildings under seismic waves, mechanical structures subjected to vibrations, pendulum chains), in biomedical systems (voltage propagation and variations in neuron's membrane), etc. Solving nonlinear estimation and control problems for such systems is important for modifying their dynamics and succeeding their functioning according to specifications (Gerdes et al., 2006), (Guo and Billings, 2007), (Gugat et al., 2009), (Lions, 1992), (Maidi and Corriou, 2014), (Winkler et al., 2009), (Winkler and Lohmann, 2010), (Saadatpour and Levi, 2013).

Following the procedure for numerical solution of the nonlinear PDE of the wave-type dynamics, a set of coupled nonlinear ordinary differential equations is obtained and written in a state-space form (Pinsky, 1991). For the latter state-space description, differential flatness properties are proven. This means that all state variables and the control inputs of the state-space model can be expressed as functions of a vector of algebraic variables which constitute the flat output and of the flat output's derivatives (Boouden et al., 2011), (Fliess and Mounier, 1999), (Mounier and Rudolph, 2001), (Lévine, 2011), (Rudolph, 2003). By applying a change of coordinates (diffeomorphism) which is based on differential flatness theory it is shown that the state-space model of the wave-type PDE can be written in the linear canonical (Brunovsky) form, in which the previously noted nonlinear ordinary differential equations are now transformed into linear ones (Rigatos, 2011), (Rigatos, 2013). Next, pointwise feedback control is applied to the wave-type PDE.

Another aim of the article is to implement state-feedback control of the nonlinear wave dynamics using measurements from a small number of sensors (Rigatos, 2011),(Rigatos, 2013),(**?**). This implies that for those state vector elements of the PDE's state-space description which cannot be measured directly, state estimation with filtering methods has to be be applied. Filtering for nonlinear distributed parameter systems is again a non-trivial problem. Both observer-based and Kalman Filter-based approaches have been proposed (Guo et al., 2012), (Hidayat et al., 2011), (Salberg et al., 2010). To this end, in this paper, a new nonlinear filtering method, under the name Derivative-free nonlinear Kalman Filtering, is proposed. The filter consists of the standard Kalman Filter recursion applied to the linear equivalent state-space model of the wave PDE (Basseville and Nikiforov, 1993), (Rigatos and Tzafestas, 2007), (Rigatos and Zhang, 2009). Moreover, an inverse transformation which is based on differential flatness properties enables to obtain estimates of the state variables of the initial system's description.

2. NONLINEAR 1D WAVE-TYPE PARTIAL DIFFERENTIAL EQUATIONS

2.1 Sine-Gordon nonlinear PDE in coupled nonlinear pendula

Nonlinear 1D wave-type partial differential equations appear in models of coupled oscillators. One can consider for example the forced damped sine-Gordon equation (Saadatpour and Levi, 2013)

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$$\frac{\partial^2\phi}{\partial t^2} + c\frac{\partial\phi}{\partial t} - \frac{\partial^2\phi}{\partial x^2} + sin(\phi) = l \quad (1)$$

where c and l are constants. This type of PDE appears in many physical phenomena, such as nonlinear resonant optics and Josephson junctions, or as a dynamic model of electrons in a crystal lattice. Eq. (1) describes the motion of an array of pendula each of which is coupled to its nearest neighbors by a torsional spring with a coupling coefficient k. Each pendulum is subject to a constant torque l and to a viscous drag force with coefficient c. The angle x_i of the i-th pendulum and the vertical axis evolves according to Eq. (1).

By considering a one dimensional grid of N sample points and by computing the second-order derivative $\frac{\partial^2\phi}{\partial x^2} \simeq \frac{1}{h^2}(\phi_{i+1} - 2\phi_i + \phi_{i-1})$, one has the decomposition of the nonlinear partial differential equation into a set of nonlinear ordinary differential equations of the form $\ddot{x}_i + c\dot{x}_i + \epsilon sin(x_i) = k(x_{i+1} - 2x_i + x_{i-1}) + l \;\; i = 1, 2, \cdots$. One can also use a periodicity condition $x_{i+N} = x_i + 2\pi$. For $k = \frac{1}{h^2} = 1$, the periodicity condition reduces the system from infinite many pendula into one with N degrees of freedom

$$\begin{array}{c} \ddot{x}_1 + cx_1 + \epsilon sin(x_1) = x_2 + x_N - 2x_1 + l - 2\pi \\ \ddot{x}_2 + cx_2 + \epsilon sin(x_2) = x_3 + x_1 - 2x_2 + l \\ \cdots \\ \ddot{x}_N + cx_N + \epsilon sin(x_N) = x_1 + x_N - 2x_N + l + 2\pi \end{array} \quad (2)$$

2.2 Sine-Gordon nonlinear PDE in the model of the Josephson junction

A transmission line is considered where transverse electromagnetic waves propagate as shown in Fig. 1. The transmission line consists of inductors, capacitors and Josephson junctions, such that for a length dx of the transmission line the capacitance is $dC = Cdx$, the inductance is $dL = Ldx$ and the critical current is $dI_0 = I_0dx$ (Saadatpour and Levi, 2013). Dividing the differential voltage drop dV and the shunt current dI by dx, one obtains

$$\begin{array}{c} \frac{\partial V}{\partial x} = -L\frac{\partial I}{\partial t} \\ \frac{\partial I}{\partial x} = -C\frac{\partial V}{\partial t} - I_0 sin(\phi) \end{array} \quad (3)$$

where ϕ is the difference $\phi = \phi_{upper} - \phi_{lower}$ of the superconducting phases. The voltage is related to the rate of change of ϕ through the Josepshon equation $\frac{\partial\phi}{\partial t} = \frac{2eV}{h}$, and equivalently $\frac{\partial\phi}{\partial x} = -\frac{2eV}{h}I$. Thus, one arrives at the equation

$$\frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} - \frac{\partial^2\phi}{\partial x^2} + \frac{1}{\lambda_j^2}sin(\phi) = 0 \quad (4)$$

where $c = (LC)^{-1/2}$ is the Swihart velocity and $\lambda_j = (\hbar/2eLI_0)^{1/2}$ is the Josephson length. By rescaling lenghts λ_j and λ_j/c one arrives at the sine-Gordon equation

$$\frac{\partial^2\phi}{\partial t^2} + c\frac{\partial\phi}{\partial t} - \frac{\partial^2\phi}{\partial x^2} + sin(\phi) = l \quad (5)$$

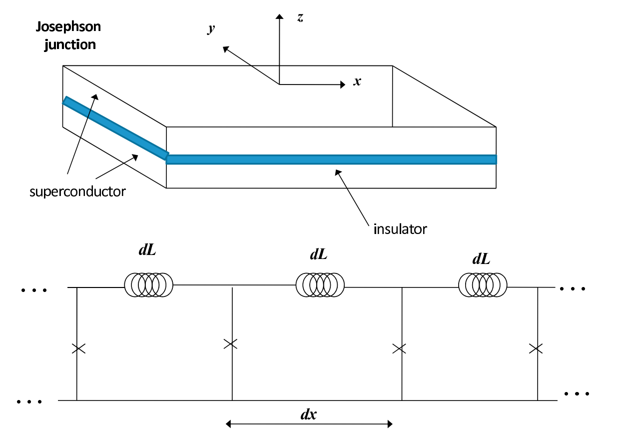

Fig. 1. Josephson transmission line described by Josephson junctions (top), a capacitance per unit length C, an inductance per unit length L and a critical current per unit length I_0

3. STATE-SPACE DESCRIPTION OF THE NONLINEAR WAVE DYNAMICS

The following 1D nonlinear wave equation (e.g. sine-Gordon dynamics) is considered next

$$\frac{\partial^2\phi}{\partial t^2} = k\frac{\partial^2\phi}{\partial x^2} + f(\phi) + u(\phi, t) \quad (6)$$

where $u(x,t)$ is assumed to be the external pointwise control input. Using the approximation for the partial derivative $\frac{\partial^2\phi}{\partial x^2} \simeq \frac{\phi_{i+1} - 2\phi_i + \phi_{i-1}}{\Delta x^2}$, and considering spatial measurements of variable ϕ along axis x at points $x_0 + i\Delta x, \; i = 1, 2, \cdots, N$ one has

$$\frac{\partial^2\phi_i}{\partial t^2} = \frac{K}{\Delta x^2}\phi_{i+1} - \frac{2K}{\Delta x^2}\phi_i + \frac{K}{\Delta x^2}\phi_{i-1} + f(\phi_i) + u(x_i, t) \quad (7)$$

By considering the associated samples of ϕ given by $\phi_0, \phi_1, \cdots, \phi_N, \phi_{N+1}$ one has

$$\begin{array}{l} \frac{\partial^2\phi_1}{\partial t^2} = \frac{K}{\Delta x^2}\phi_2 - \frac{2K}{\Delta x^2}\phi_1 + \frac{K}{\Delta x^2}\phi_0 + f(\phi_1) + u(\phi_1, t) \\ \frac{\partial^2\phi_2}{\partial t^2} = \frac{K}{\Delta x^2}\phi_3 - \frac{2K}{\Delta x^2}\phi_2 + \frac{K}{\Delta x^2}\phi_1 + f(\phi_2) + u(\phi_2, t) \\ \frac{\partial^2\phi_3}{\partial t^2} = \frac{K}{\Delta x^2}\phi_4 - \frac{2K}{\Delta x^2}\phi_3 + \frac{K}{\Delta x^2}\phi_2 + f(\phi_3) + u(\phi_3, t) \\ \cdots \\ \frac{\partial^2\phi_N}{\partial t^2} = \frac{K}{\Delta x^2}\phi_{N+1} - \frac{2K}{\Delta x^2}\phi_N + \frac{K}{\Delta x^2}\phi_{N-1} + f(\phi_N) + u(\phi_N, t) \end{array} \quad (8)$$

By defining the following state vector $x^T = (\phi_1, \phi_2, \cdots, \phi_N)$, one obtains the following state-space description

$$\begin{array}{l} \ddot{x}_1 = \frac{K}{\Delta x^2}x_2 - \frac{2K}{\Delta x^2}x_1 + \frac{K}{\Delta x^2}\phi_0 + f(x_1) + u(x_1) \\ \ddot{x}_2 = \frac{K}{\Delta x^2}x_3 - \frac{2K}{\Delta x^2}x_2 + \frac{K}{\Delta x^2}x_1 + f(x_2) + u(x_2) \\ \ddot{x}_3 = \frac{K}{\Delta x^2}x_4 - \frac{2K}{\Delta x^2}x_3 + \frac{K}{\Delta x^2}x_2 + f(x_3) + u(x_3) \\ \cdots \\ \ddot{x}_N = \frac{K}{\Delta x^2}\phi_{N+1} - \frac{2K}{\Delta x^2}x_N + \frac{K}{\Delta x^2}x_{N-1} + f(x_N) + u(x_N) \end{array} \quad (9)$$