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Merging Kalman Filtering and Zonotopic State Bounding for Robust Fault Detection under Noisy Environment

C. Combastel *

* ECS-Lab (EA n° 3649) - ENSEA, 6 avenue du Ponceau, 95014 Cergy-Pontoise Cedex, France (e-mail: combastel@ensea.fr).

Abstract: A joint Zonotopic and Gaussian Kalman Filter (ZGKF) is proposed for the robust fault detection of discrete-time LTV systems simultaneously subject to bounded disturbances and Gaussian noises. Given a maximal probability of false alarms, a detection test is developed and shown to merge the usually mutually exclusive benefits granted by set-membership techniques (robustness to worst-case within specified bounds, domain computations) and stochastic approaches (taking noise distribution into account, probabilistic evaluation of tests). The computations remain explicit and can be efficiently implemented. A numerical example illustrates the improved tradeoff between sensitivity to faults and robustness to disturbances/noises.

Keywords: Bounded disturbances, Random noise, Robust estimation, Fault detection, Uncertain dynamic systems, Kalman filters, Intervals, Sets, Zonotopes, Gaussian processes.

1. INTRODUCTION

The design of a reliable fault diagnosis heavily relies on relevant descriptions of disturbances and noises. Indeed, not only a nominal trajectory but also a whole set of possible deviations from it must be characterized as accurately as possible for a faulty behavior to be almost surely detected in spite of uncertainties which are inherent to the modeling task. Either mainly based on knowledge or data, the models used for fault detection and diagnosis (Frank (1990); Blanke et al. (2003); Isermann (2005); Ding (2008)) remain subject to such requirements to achieve a good tradeoff between sensitivity to faults and robustness to disturbance and noises. When dealing with uncertainties, two usually distinct paradigms can be used: the stochastic one and the set-membership (or bounded-error) one. Based on stochastic processes, Kalman filtering (Kalman (1960); Maybeck (1979)) has been successfully used in a wide range of applications, including fault detection. Mainly based on Gaussian probability distributions (in spite of several kinds of extensions), it appears to be often well suited to deal with measurement noises. However, the modeling of disturbances mostly related to some lack of knowledge about deterministic behaviors (e.g. load torque of a motor under incompletely specified operating conditions) is often more representative using bounded errors than Gaussian distributions. Indeed, such disturbances can successively vary arbitrarily, then temporarily remain constant but equal to unknown values, then vary again but differently, etc., and do not have any other stationary behavior than that of remaining within specified bounds, at least under fault-free operation. Set-membership techniques, either based on ellipsoids (Schweppe (1968); Maksarov and Norton (2002); Kurzhanskiy and Varaiya (2007)), intervals (Moore (1966); Jaulin et al. (2001); Raïssi et al. (2012)), polytopes or zonotopes (Kühn (1998a); Puig et al. (2003); Combastel (2003, 2015)) are well suited to deal with them. In this context, state bounding observers based on predictor/corrector approaches (so resembling Kalman filters) can be used (Jaulin et al. (2001); Combastel (2003)). However, contrary to stochastic Kalman filters which efficiently deal with random (measurement) noises, a bounded-error description of measurement noise often induces an unnecessary loss of precision reducing the sensitivity to faults.

Based on a discrete-time LTV fault-free model simultaneously excited by bounded disturbances and Gaussian noises, the contribution of this paper is to propose a computationally efficient solution merging stochastic Kalman filtering and zonotopic state bounding for robust fault detection under noisy environment: a detection test derived from the so-called ZGKF algorithm is developed to combine the usually mutually exclusive benefits granted by set-membership techniques (robustness to worst-case within specified bounds, domain computations) and stochastic approaches (taking noise distribution into account, probabilistic evaluation of tests e.g. false alarm rates).

The paper is organized as follows: after preliminaries in section 2, the problem formulation is given in section 3. Based on the observer structure detailed in section 4, a multi-objective optimality criterion is proposed in section 5. Its purpose is to compute an optimal observer gain (section 6) leading to the ZGKF algorithm given in section 7. Based on explicit confidence domains, a fault detection test satisfying a requirement in terms of probability of false alarms is proposed in section 8 and illustrated through a numerical example in section 9.

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2. PRELIMINARIES

2.1 Probabilities: definitions and notations

Let $\mathfrak{P} = (\Omega, \Sigma, \mathcal{P})$ be a probability space, where Ω is a set of possible outcomes, $\Sigma \subset 2^{\Omega}$ (powerset of Ω) defines a collection of events, and $\mathcal{P}$ is a probability measure. Let $\mathbf{x}$ and $\mathbf{y}$ be two random real vectors defined on $\mathfrak{P}$. Boldface names denote random variables. The expectation of $\mathbf{x}$ is $\mathrm{E}[\mathbf{x}] = \int_{\Omega} \mathbf{x} d\mathcal{P}$. E[.] is a linear operator. The (cross)covariance between $\mathbf{x}$ and $\mathbf{y}$ is: $\mathrm{Cov}(\mathbf{x}, \mathbf{y}) = \mathrm{E}[(\mathbf{x} - \mathrm{E}[\mathbf{x}])(\mathbf{y} - \mathrm{E}[\mathbf{y}])^T] = \mathrm{E}[\mathbf{x}\mathbf{y}^T] - \mathrm{E}[\mathbf{x}]\mathrm{E}[\mathbf{y}]^T$. The covariance of $\mathbf{x}$ (or variance in the scalar case) is $\mathrm{Cov}(\mathbf{x}) = \mathrm{Cov}(\mathbf{x}, \mathbf{x})$. For continuous random vectors like $\mathbf{x}$, a probability density function (pdf), $\rho_{\mathbf{x}} : \mathbb{R}^n \to \mathbb{R}$ is such that $\forall \mathcal{D} \subset \mathbb{R}^n$, $\mathcal{P}(\mathbf{x} \in \mathcal{D}) = \int_{\mathcal{D}} \rho_{\mathbf{x}}(x) dx$, where $\mathcal{P}(\mathbf{x} \in \mathcal{D})$ is the probability that an outcome leads $\mathbf{x}$ to fall inside the domain $\mathcal{D}$. The support $\mathcal{S}_{\mathbf{x}}$ of $\mathbf{x}$ is the smallest closed set whose complement has probability zero. So, $\mathcal{P}(\mathbf{x} \in \mathcal{S}_{\mathbf{x}}) = 1$.

2.2 Gaussian distribution and confidence ellipsoids

Let $\mathbf{x} \sim \mathcal{N}(c, Q)$ refer to a random vector following a Gaussian (normal) probability distribution with center $c \in \mathbb{R}^n$ and covariance matrix $Q \in \mathbb{R}^{n \times n}$:

$$\rho_{\mathbf{x}}(x) = \frac{1}{\sqrt{(2\pi)^n \det(Q)}} \exp(-\frac{1}{2}(x-c)^T Q^{-1}(x-c)). \quad (1)$$

The support $\mathcal{S}_{\mathbf{x}} = \mathbb{R}^n$ is unbounded, but a confidence ellipsoid $(c, Q)_\alpha$ can be defined as:

$$(c, Q)_\alpha = \{x \in \mathbb{R}^n,\ (x-c)^T Q^{-1}(x-c) \leq \chi_n^2(1-\alpha)\}, \quad (2)$$

where $\chi_n^2(1-\alpha) \in \mathbb{R}$ is the value taken for probability $1-\alpha$ by the quantile function of the chi-squared distribution with n degrees of freedom. α can be interpreted as a probability of type I error (false alarm rate) when testing the membership of an outcome of $\mathbf{x}$ to $(c, Q)_\alpha$:

$$\mathbf{x} \sim \mathcal{N}(c, Q) \Rightarrow \mathcal{P}(\mathbf{x} \in (c, Q)_\alpha) = 1 - \alpha. \quad (3)$$

2.3 Zonotopes

A zonotope $\langle c, R \rangle \subset \mathbb{R}^n$ with center $c \in \mathbb{R}^n$ and generator matrix $R \in \mathbb{R}^{n \times p}$ is a polytopic set defined as the linear image of the unit hypercube $[-1, +1]^p$ by R:

$$\langle c, R \rangle = \{c + Rs,\ \|s\|_\infty \leq 1\}. \quad (4)$$

$\langle R \rangle = \langle 0, R \rangle$ is called centered zonotope. Any permutation of the columns of R leaves it invariant. The Minkowski sum of two sets S_1 and S_2 is $S_1 \oplus S_2 = \{s_1 + s_2,\ (s_1, s_2) \in S_1 \times S_2\}$. The linear image of the set $S \subset \mathbb{R}^n$ by $L \in \mathbb{R}^{q \times n}$ is $L \odot S = \{Ls,\ s \in S\}$. Zonotopes form a class of polytopic sets implicitly represented by matrices and leading to efficient set computations. This class is closed under Minkowski sum $\oplus$ (computed as a matrix concatenation) and linear image $\odot$ (computed as a matrix product):

$$\langle c_1, R_1 \rangle \oplus \langle c_2, R_2 \rangle = \langle c_1 + c_2, [R_1, R_2] \rangle, \quad (5)$$

$$L \odot \langle c, R \rangle = \langle Lc, LR \rangle. \quad (6)$$

$$\langle c, R \rangle \subset \langle c, b(R) \rangle, \quad b(R) = \mathrm{diag}(|R|\mathbf{1}), \quad (7)$$

(7) shows how a zonotope $\langle c, R \rangle$ can be enclosed within an aligned box (or interval hull) defined by $b(R) \in \mathbb{R}^{n \times n}$, where $|.|$ is the element-by-element absolute value operator, $\mathbf{1}$ is a column vector of ones, and diag(.) returns a diagonal matrix from a vector defining the diagonal elements (or reciprocally). Such a box enclosure usually being too conservative, a reduction operator can be used: Kühn (1998b). In this work, the reduction operator first introduced in Combastel (2003), improved in Combastel (2005), and further used in several works e.g. Alamo et al. (2005); Girard (2005); Althoff et al. (2010); Montes de Oca et al. (2012); Le et al. (2013) will be denoted $\downarrow_q$, where q specifies the maximum number of columns of matrix $\downarrow_q R$ satisfying the inclusion property $\langle R \rangle \subset \langle \downarrow_q R \rangle$. The reduction operator $\downarrow_q$ illustrated in Fig. 1 first consists in sorting the columns of $R \in \mathbb{R}^{n \times p}$ on decreasing Euclidean norm $\|.\|_2$ (8) and enclosing the $p - q + n$ smaller columns (generator segments) into a box:

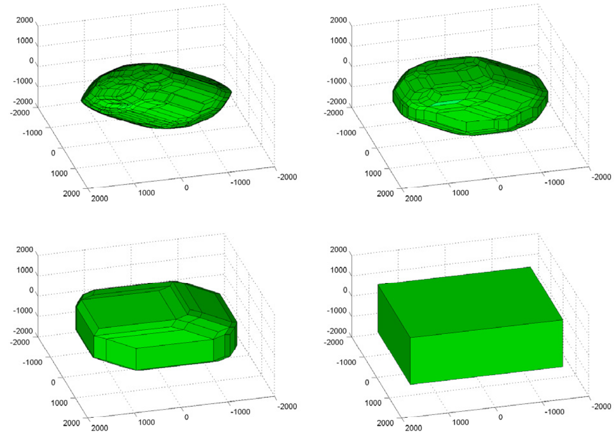

Fig. 1. From left to right and top to bottom, a 3D zonotope $\langle R \rangle$ generated by 30 segments ($R \in \mathbb{R}^{3 \times 30}$) and some of its reductions with decreasing complexity: $\langle \downarrow_{20} R \rangle$, $\langle \downarrow_{10} R \rangle$, and $\langle \downarrow_3 R \rangle = \langle b(R) \rangle$ (interval hull of $\langle R \rangle$).

$$R = [r_1, \ldots, r_j, \ldots, r_p], \quad \|r_j\|_2 \geq \|r_{j+1}\|_2, \quad (8)$$

$$\text{If } p \leq q, \text{ Then } \downarrow_q R = R, \text{ Else } \downarrow_q R = [R_>, b(R_<)], \quad (9)$$

$$\text{with } R_> = [r_1, \ldots, r_{q-n}],\ R_< = [r_{q-n+1}, \ldots, r_p]. \quad (10)$$

Let $\mathbf{x} \sim \mathcal{Z}(c, R)$ refer to a random vector following a probability distribution such that the support $\mathcal{S}_{\mathbf{x}}$ of $\mathbf{x}$ is included in the zonotope $\langle c, R \rangle$. Then:

$$\mathbf{x} \sim \mathcal{Z}(c, R) \Rightarrow \mathcal{P}(\mathbf{x} \in \langle c, R \rangle) = 1. \quad (11)$$

Definition 1. (Covariation). The covariation of $\langle c, R \rangle$ is defined as $\mathrm{cov}(\langle c, R \rangle) = RR^T$. By extension, the covariation of $\mathbf{x} \sim \mathcal{Z}(c, R)$ is $\mathrm{cov}(\mathbf{x}) = RR^T$.

Definition 2. (F-radius). The F-radius of $\langle c, R \rangle$ is the Frobenius norm of R: $\|R\|_F = \mathrm{tr}(R^T R) = \mathrm{tr}(RR^T)$. By extension, the F-radius of $\mathbf{x} \sim \mathcal{Z}(c, R)$ is $\mathrm{tr}(\mathrm{cov}(\mathbf{x}))$.

Given $R \in \mathbb{R}^{n \times p}$ as in (8), the F-radius of $\langle c, R \rangle$ equals $\sum_{j=1}^{p} \|r_j\|_2^2$ i.e. the sum of the squared lengths of all the generator segments of $\langle c, R \rangle$. The F-radius thus interprets as a size criterion for zonotopes.

2.4 Matrix calculus

$\partial_X f(X)$ is a short notation for $\partial f(X)/\partial X$. If the function f returns scalar values and $X = [X_{ij}]$ is a matrix, then $\partial_X f(X) = [\partial_{X_{ji}} f(X)]$. tr(.) denoting the trace of a square matrix ($\mathrm{tr}(A) = \sum_i A_{ii}$), and X, A, B, C being matrices of appropriate size, it comes: