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Novel cubature Kalman filtering for systems involving nonlinear states and linear measurements

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ABSTRACT

This paper extends the cubature Kalman filter (CKF) to deal with systems involving nonlinear states and linear measurements (herein called the nonlinear–linear combined systems) with additive noise. The method is referred to as the nonlinear–linear square-root cubature Kalman filtering (NL-SCKF). In NL-SCKF, the cubature rule, combined with a QR decomposition, singular value decomposition and a linear update without requirement of cubature points, is designed to update nonlinear states and linear measurements. In addition, the convergence analysis of NL-SCKF is performed. Simulation results in two selected problems, namely filtering chaotic signals and chaos-based communications, indicate that the proposed NL-SCKF with lower computation complexity achieves the same accuracy as the standard SCKF, and outperforms CKF significantly.

1. Introduction

The Bayesian filtering provides a unified framework based on a recursive approach to solving nonlinear filtering problems. In general, the Bayesian filtering is developed on the basis of a state-space model which consists of two components, namely, a process model describing the evolution of a hidden state of the system, and a measurement model capturing noisy observables related to the hidden state [1]. In the Bayesian filter, the posterior density of the system's state provides a complete statistical description for the state variables at a given time. In practice, a suboptimal solution of the posterior density is employed either locally or globally. In the local approach, the predicted density and the posterior density are assumed to be Gaussian. Nonlinear filters, such as the extended Kalman filter (EKF) [2,3], the central-difference Kalman filter (CDKF) [4], the unscented Kalman filter (UKF) [5,6], and the quadrature Kalman filter (QKF) [7–10], fall under this category. However, these filters may suffer from divergence or the curse of dimensionality [8,12]. In the global approach, e.g., the particle filter (PF), no assumption is made about the posterior density, which is approximated by a large number of samples [13]. Typically, the global approaches involve enormous computational overheads, and are more difficult to implement in practice.

Recently, the third-degree spherical-radial cubature rule (SRCR) and spherical simplex-radial cubature rule (SSRCR) are applied to numerically compute multivariate moment integrals involved in the nonlinear Bayesian filter. This has formed the basis of the cubature Kalman filter (CKF) [1,11,12,14] and the spherical simplex-radial cubature Kalman filter (SSRCKF) [15,16]. The SRCR method only requires an even number of equally weighted cubature points ($2n$ points, where n is the dimension of the state vector) distributed uniformly over an ellipsoid centered at the origin [17], yet the SSRCR method requires $2n+2$ cubature points [15]. In addition, at the cost of high computational complexity, the corresponding high-order CKF and SSRCKF improve numerical accuracy compared with CKF and SSRCKF to some extent. Hence, compared with the aforementioned local approaches, CKF provides an efficient nonlinear filter that could be applied to high-dimensional nonlinear filtering problems with minimal computational effort.

The design of CKF is based on a dynamical state-space model that describes the physical process and measurement equations. In practice, many nonlinear filtering problems can be described by nonlinear process equations and linear measurement equations [3]. It is therefore of practice interest to develop a novel CKF for nonlinear–linear combined systems to improve computation efficiency. Chaos-based spread spectrum communication [18,19] and noise reduction of chaotic signals [20] are two good examples of

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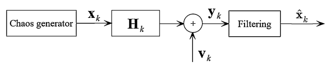

Fig. 1. Block diagram of filtering noisy contaminated chaotic signal.

nonlinear–linear combined systems. The motivation of this paper is to extend CKF to deal with the aforementioned class of systems involving nonlinear states and linear measurements, and to perform its convergence analysis. The resulting filtering algorithm is here referred to as the nonlinear–linear square-root cubature Kalman filtering (NL-SCKF).

The rest of the paper is organized as follows. Section 2 presents a general nonlinear filtering problem in the chaotic signal context. In Section 3, the new NL-SCKF is developed by using SVD and the related matrix operations based on CKF. The convergence analysis of the proposed NL-SCKF is performed in Section 4. Section 5 provides computer simulations, demonstrating the convergence features and efficiency of NL-SCKF. An application to chaotic parameter modulation is also described. Further development of NL-SCKF is discussed in Section 6. Section 7 concludes this paper.

2. Problem statement

We consider the problem of filtering chaotic signals with additive Gaussian noise as shown in Fig. 1, which can be described by the following dynamical state-space model in discrete time.

$$\mathbf{x}_k = \mathbf{f}(\mathbf{x}_{k-1}) \tag{1}$$

$$\mathbf{y}_k = \mathbf{H}_k\mathbf{x}_k + \mathbf{v}_k, \tag{2}$$

where $\mathbf{x}_k \in \mathbb{R}^n$ is the state vector of the nonlinear dynamical system in discrete time k; $\mathbf{f} : \mathbb{R}^n \to \mathbb{R}^n$ denotes a nonlinear process equation for generating chaotic signals; $\mathbf{H}_k$ is the $m \times n$ measurement matrix; $\mathbf{y}_k \in \mathbb{R}^m$ is the measurement vector; and $\mathbf{v}_k$ is the measurement Gaussian noise with zero mean and covariance $\mathbf{R}_k$.

The nonlinear–linear combined system has already been applied in control, navigation, chaotic communication and so on [18,19,21]. Suppose the predictive density $p(\mathbf{x}_k|\mathbf{y}_{1:k-1})$ and the filter likelihood density $p(\mathbf{y}_k|\mathbf{y}_{1:k})$ are both Gaussian. The Gaussian posterior density $p(\mathbf{x}_k|\mathbf{y}_{1:k})$ is consequentially Gaussian. In the Bayesian filtering paradigm, the posterior density of the state provides a complete statistical description of the state at a given time [22]. Therefore, the solution to the posterior density constitutes a state estimation [11], i.e.,

$$\hat{\mathbf{x}}_k = \int_{\mathbb{R}^n} \mathbf{f}(\mathbf{x}_{k-1})p(\mathbf{x}_k|\mathbf{y}_{1:k})d\mathbf{x}_k. \tag{3}$$

The estimation of the state vector $\mathbf{x}_k$ in Bayesian filter reduces to computing multi-dimensional integrals. To avoid directly solving the above integral (3), numerical integration methods are required. Consider the posterior density having a standard Gaussian density with zero mean and unit covariance. An approximation $I(\mathbf{f})$ of this multi-dimensional integral (3) can be implemented by an L-point numerical integration, i.e.,

$$I(\mathbf{f}) \approx \sum_{i=1}^{L} \omega_i \mathbf{f}(\boldsymbol{\zeta}_i), \tag{4}$$

where $\boldsymbol{\zeta}_i$ is a set of points and ω_i is the associated weight.

The third-degree spherical-radial cubature rule entails a total of $2n$ cubature points for computing the integral [17]. The cubature points and their associated weights for computing a standard Gaussian weighted integral are shown as follows [11,17]:

$$\boldsymbol{\zeta}_i = \sqrt{n}\left\{ \begin{pmatrix} 1 \\ \vdots \\ 0 \end{pmatrix}, \ldots, \begin{pmatrix} 0 \\ \vdots \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ \vdots \\ 0 \end{pmatrix}, \ldots, \begin{pmatrix} 0 \\ \vdots \\ -1 \end{pmatrix} \right\} \tag{5}$$

$$\omega_i = \frac{1}{L}, \quad i = 1, 2, \ldots, \quad L = 2n. \tag{6}$$

Therefore, the integral in (3) can be computed by the cubature-point set $\{\boldsymbol{\zeta}_i, \omega_i\}$ based on (4). The cubature Kalman filter (CKF) can be incorporated in the third-degree spherical-radial cubature rule of the Bayesian filter to provide a systematic solution to high dimensional nonlinear filtering problems.

3. Nonlinear–linear square-root cubature Kalman filter

In the standard CKF algorithm, the error covariance matrix may lose symmetry and positive definiteness, causing unstable or even divergence behavior [11,23]. To circumvent this problem, a square-root cubature Kalman filter (SCKF) [11] is introduced to propagate the square root $\mathbf{A}$ of error covariance $\mathbf{P}$, and the square root can also preserve the symmetry and positive definiteness of the covariance matrix for improving numerical stability of CKF.

The QR decomposition of $\mathbf{A}$ is used to keep the square root as a triangular matrix for computational convenience [7,24], i.e.,

$$\mathbf{P} = \mathbf{A}\mathbf{A}^T = \mathbf{R}^T\mathbf{Q}^T\mathbf{Q}\mathbf{R} = \mathbf{R}^T\mathbf{R} = \mathbf{S}\mathbf{S}^T, \tag{7}$$

where $\mathbf{A}^T = \mathbf{QR}$, $\mathbf{S} = \mathbf{R}^T$, and $\mathbf{R}$ is an upper triangular matrix. Thus, $\mathbf{S}$ is also a triangular matrix, and its sparseness reduces storage space and improves computational efficiency. The QR decomposition of the square root of the error covariance in CKF constitutes the key feature of SCKF.

In addition, matrix inversion is a main numerically sensitive operation of CKF, which can also destroy symmetry and positive definiteness of the covariance matrix [11,23]. Singular value decomposition can be used to calculate the matrix inverse efficiently, especially in the case of square matrices. Factorize the square root $\mathbf{A}$ of error covariance $\mathbf{P}$ using the SVD operation, i.e., $\mathbf{A} = \mathbf{UDV}^T$, where $\mathbf{U}$ and $\mathbf{V}$ are both orthogonal matrices, and $\mathbf{D}$ is the matrix filled with zeros everywhere except along the main diagonal of its maximal upper-left square submatrix. Hence, the error covariance $\mathbf{P}$ can be rewritten as

$$\mathbf{P} = \mathbf{A}\mathbf{A}^T = \mathbf{UDV}^T\mathbf{VD}^T\mathbf{U}^T = \mathbf{UDD}^T\mathbf{U}^T, \tag{8}$$

According to the characteristics of $\mathbf{U}$ and $\mathbf{D}$, the matrix inverse of $\mathbf{P}$ can be calculated by

$$\mathbf{P}^{-1} = \mathbf{U}(\mathbf{DD}^T)^{-1}\mathbf{U}^T, \tag{9}$$

where $(\mathbf{DD}^T)^{-1}$ can be obtained easily by only calculating the reciprocal of the square of the main diagonal of $\mathbf{D}$.

Based on the nonlinear process equation (1) and the linear measurement equation (2) in the presence of additive Gaussian noise, a set of third-degree spherical-radial cubature points are sampled for computing the predictive density in the time update, and the linear Kalman filter is used in the measurement update for alleviating the computational burden. Using SCKF and SVD, we propose a novel nonlinear–linear square-root cubature Kalman filtering algorithm, which is described as follows.

Suppose the square-roots of the two covariance matrices satisfy $\mathbf{P}_k = \mathbf{S}_k\mathbf{S}_k^T$ and $\mathbf{P}_{k|k-1} = \mathbf{S}_{k|k-1}\mathbf{S}_{k|k-1}^T$. The cubature-point set $\{\boldsymbol{\zeta}_i, \omega_i\}$ is calculated using (5) and (6). The NL-SCKF algorithm for estimating the state vectors $\mathbf{x}_k$ in the nonlinear–linear combination system's state-space model (1) and (2) can be summarized as follows.