



A Quadratic Kalman Filter^{☆,☆☆}



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ABSTRACT

We propose a new filtering and smoothing technique for non-linear state-space models. Observed variables are quadratic functions of latent factors following a Gaussian VAR. Stacking the vector of factors with its vectorized outer-product, we form an augmented state vector whose first two conditional moments are known in closed-form. We also provide analytical formulae for the unconditional moments of this augmented vector. Our new Quadratic Kalman Filter (QKF) exploits these properties to formulate fast and simple filtering and smoothing algorithms. A simulation study first emphasizes that the QKF outperforms the extended and unscented approaches in the filtering exercise showing up to 70% RMSEs improvement of filtered values. Second, it provides evidence that QKF-based maximum-likelihood estimates of model parameters always possess lower bias or lower RMSEs than the alternative estimators.

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1. Introduction

This paper proposes a new discrete-time Kalman filter for state-space models where the transition equations are linear and the measurement equations are quadratic. We call this method the *Quadratic Kalman Filter* (QKF). While this state-space model has become increasingly popular in the applied econometric literature, existing filters are either highly computationally intensive, or not specifically fitted to the linear-quadratic case. We begin by building the augmented vector of factors stacking together the latent vector and its vectorized outer-product. To the best of our knowledge, this paper is the first to derive analytically and provide closed-form formulae of both the conditional and

the unconditional first-two moments of this augmented vector.¹ Using these moments, the transition equations of the augmented vector are expressed in an affine form. Similarly, the measurement equations are rewritten as affine functions of the augmented vector of factors. We thus obtain an *augmented state-space model* that is fully linear.

We perform the derivation of the QKF filtering and smoothing algorithms by applying the linear Kalman algorithms to the augmented state-space model. To do so, we approximate the conditional distribution of the augmented vector of factors given its own past by a multivariate Gaussian distribution. Since no adaptation of the linear algorithm is needed, the QKF combines simplicity of implementation and computational speed. We apply the same method for the derivation of the Quadratic Kalman Smoothing algorithm (QKS). Since the QKF and QKS require no simulations, it represents a convenient alternative to particle filtering.

To compare our filter with the popular existing *traditional filters* (see Tanizaki, 1996), namely the first- and second-order extended and the unscented Kalman filters, we implement a

[☆] Functions for the Quadratic Kalman Filter are implemented with the R-software and are available on the [runmycode](http://www.runmycode.org/companion/view/313)-website at <http://www.runmycode.org/companion/view/313>.

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¹ Buraschi et al. (2008) provide formulae of conditional first-two moments for the specific case of centered Wishart processes.

Monte-Carlo experiment. In order to explore a broad range of cases, we build a benchmark state-space model with different values for (i) the persistence of the latent process, (ii) the importance of noise variance in the observable, and (iii) the importance of quadratic terms in the observables. Root mean squared error (RMSE) measures are computed in each case. We compare the filters with respect to two different criteria: filtering, i.e. retrieving latent factors precisely from a fixed set of parameters, and parameter estimation, i.e. the capacity to estimate the state-space model parameters.

First, these computations provide evidence of the superiority of the QKF filtering over its competitors in all cases. When the measurement equations are fully quadratic, the QKF is the only filter able to capture the non-linearities and to produce time-varying evaluations of the latent factors. This results in up to 70% lower RMSEs for the QKF compared to the other filters, all cases considered. For measurement equations with both linear and quadratic terms, the QKF still results – to a smaller extent – in lower filtering RMSEs. These results are robust to the persistence degree of the latent process and the size of the measurement noise. Also, we emphasize that the first-order extended Kalman filter performs particularly poorly in some cases and should therefore be discarded for filtering in the linear–quadratic state-space model.

Second, the QKF-based maximum-likelihood estimates of model parameters always possess lower bias or lower RMSEs than the alternative estimators. We provide evidence that this superiority is robust to the degree of persistence of the latent process, to the degree of linearity of the measurement equations, and to the size of the measurement errors. We conclude that the QKF results in the best bias/variance trade-off for quasi maximum likelihood estimations.

The remainder of the paper is organized as follows. Section 2 provides a brief review of the non-linear filtering literature and its applications. Section 3 presents the state-space model and builds the QKF. Section 4 performs a comparison of the QKF with popular competitors using Monte-Carlo experiments. Section 5 concludes. Proofs are gathered in the Appendices.

2. Literature review

The existing *traditional* non-linear filters use linearization techniques to transform the state-space model. First and second-order extended Kalman filters build respectively on first and second-order Taylor expansions of transition and measurement equations. The first-order extended Kalman filter is extensively covered in Anderson and Moore (1979). To reduce the errors linked to the first-order approximations, Athans et al. (1968) develop a second-order extended Kalman filter.² In the general non-linear case, both methods require numerical approximations of gradients and Hessian matrices, potentially increasing the computational burden.³ The unscented Kalman filter belongs to the class of *deterministic density estimators*, and was originally implemented as an alternative to the previous techniques for applications in physics. It is a derivative-free method which is shown to be computationally close to the second-order extended Kalman filter in terms of complexity (see Julier et al., 2000 or Julier and Uhlmann, 2004).⁴ Whereas many other filters exist, the extended and unscented filters have been the most widely used in recent econometric applications.

² This method is treated in continuous and continuous-discrete time in Maybeck (1982). Bar-Shalom et al. (2002) propose a complete description of this second-order filter.

³ Gustafsson and Hendeby (2012) build a derivative-free version of the second-order extended Kalman filter which avoids issues due to numerical approximations, but shows a similar computational complexity.

⁴ A general version of the algorithm is provided in the web supplement (see Appendix B).

We consider here a specification in which the transition equations are affine and the measurement equations are quadratic. This quadratic framework is particularly suited to numerous dynamic economic models. While first-order linearization is standard and largely employed in the dynamic stochastic general equilibrium (DSGE) literature, the algorithm we develop is fitted to exploit second-order approximations.⁵ As for finance, an important field of application of our filter is the modeling of term structures of interest rates.⁶ The standard and popular Gaussian affine term-structure model (GATSM) provides yields which are affine combinations of dynamic linear auto-regressive factor processes. These models often include latent factors and, to estimate them, the linear Kalman filter⁷ has gained overwhelming popularity compared to other techniques (see e.g. Duan and Simonato, 1999 or Joslin et al., 2011). A natural extension of the GATSM is to assume that yields are quadratic functions of factor processes. The bulk of the papers using QTSMs considers the dynamics of government-bond yield curves (e.g. Leippold and Wu, 2007 and Kim and Singleton, 2012). QTSMs have also been shown to be relevant to model the dynamics of positive risk intensities and their implied term structures: while default intensities are considered in the credit-risk literature (see e.g. Doshi et al., 2013 and Dubecq et al., 2013), mortality intensities have also been modeled in this framework (Gourieroux and Monfort, 2008). In order to estimate QTSMs involving latent variables, a wide range of techniques are considered in the existing literature: Inci and Lu (2004) and Li and Zhao (2006) use the extended Kalman filter, Leippold and Wu (2007), Doshi et al. (2013) or Chen et al. (2008) employ the unscented Kalman filter and Andreasen and Meldrum (2011) opt for the particle filter.⁸ Finally, Dubecq et al. (2013) use the QKF filter that is developed hereafter. The quadratic state-space framework that we consider in the present paper is also well-suited to work with Wishart processes. These processes have been used in various empirical-finance studies. In most cases, they are employed in multivariate stochastic volatility models (see e.g. Jin and Maheu, 2013 or Rinnergschwentner et al., 2011). Wishart processes have also been exploited in several QTSMs (Filipovic and Teichmann, 2002; Gourieroux et al., 2010; Gourieroux and Sufana, 2011).

3. The Quadratic Kalman Filter (QKF) and Smoother (QKS)

3.1. Model and notations

We are interested in a state-space model with affine transition equations and quadratic measurement equations. We consider the following model involving a latent (or state) variable X_t of size n and an observable variable Y_t of size m . X_t might be only partially latent, that is, some components of X_t might be observed.

Definition 3.1. The linear–quadratic state-space model is defined by:

$$X_t = \mu + \Phi X_{t-1} + \Omega \varepsilon_t \quad (1a)$$

⁵ Our approach could for instance be exploited to estimate the standard asset-pricing model of Burnside (1998) considered e.g. by Collard and Juillard (2001).

⁶ See Dai and Singleton (2003) for a survey of interest-rate term-structure modeling literature.

⁷ See Kalman (1960) for the original linear filter derivation. Properties are developed in e.g. Harvey (1991) or Durbin and Koopman (2012).

⁸ Ahn et al. (2002) resort to the efficient method of moments (EMM). However, Duffee and Stanton (2008) show that, compared to maximum likelihood approaches, EMM has poor finite sample properties when data are persistent, a typical characteristic of bond yields. Moreover, while EMM is used to estimate model parameters, it does not directly provide estimates of the latent factors. Galant and Tauchen (1998) however propose a reprojection method to recover latent variables after having estimated the model parametrization by means of EMM.

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