Inertial/celestial-based fuzzy adaptive unscented Kalman filter with Covariance Intersection algorithm for satellite attitude determination

Wang Xiaochu, You Zheng *, Zhao Kaichun *

State Key Laboratory of Precision Measurement Technology and Instruments, Tsinghua University, Beijing 100084, China

ABSTRACT

This paper deals with the attitude determination algorithm for the satellite with an attitude measurement unit that is comprised of one gyroscope and two star trackers installed perpendicularly. Since the data updating rate of star trackers is typically lower than that of the gyro and filter, appropriate compensation is made for star sensors, but this results in more difficulties of determining the performed noise level. A fuzzy adaptive tuning method is used to help tuning, and with modified Rodrigues parameters and rotation vector to represent attitude error, a fuzzy adaptive unscented Kalman filter with minimal skew sampling method is realized, which works as a sub-system and estimates sub-optimal attitude states and gyro bias. Two such sub-systems are federated into the framework of Covariance Intersection algorithm to achieve data fusion for an optimal attitude and gyro bias estimation in system level. Simulation is performed to verify the attitude determination algorithm presented in this paper.

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1. Introduction

As a crucial method of the attitude determination system of satellites, the attitude estimation could provide more accurate results than direct measurements, enabling satellites to carry out many tasks that place high requirements on the attitude accuracy. From 1960s, numbers of studies on attitude estimation have been performed, among which extended Kalman filter (EKF) and unscented Kalman filter (UKF) are most effective. EKF for attitude estimation is fully discussed in previous research, covering nearly all the details [1–3]. But if the system has severe nonlinearities, EKF will sometimes provide bad estimates, because the first-order truncation of Taylor expansion could not approximate the nonlinearities well anymore. Since Julier and Uhlmann developed the method of unscented transformation to propagate mean and covariance information through nonlinear transformations, research focusing on UKF has been carried on [4–6]. And UKF is considered with a better capability of estimating the attitude of satellites than EKF, because it could provide better results and converge fast even if initial conditions are inaccurate.

And as the complexity of satellites and space tasks increases, there is an ever increasing need for more accurate and robust estimators. Among many attitude sensors, star trackers are most

popular sensors because they can provide arc-second attitude accuracy. It is more and more common that two star trackers are equipped in one satellite, providing data redundancy, accuracy, and reliability, such as the results shown in Refs. [7] and [8].

Even though the UKF has been standardized, there are yet still some practical problems for this to apply. Firstly, it is known that, the shorter the sampling time interval is, the better performance the estimation has, and thus the more real-time the estimation is. However, the output frequencies of some attitude sensors are not high enough to satisfy this requirement. Taking CMOS based star trackers for example, limited by the exposure time of 180 milliseconds, their data updating frequencies would be at most 5 Hz in theory. This means data outputs refresh only every 0.2 second, and during the 0.2-second period, outputs remain constant and thus loosing accuracy. So if these sensors are used in a UKF with sampling time interval shorter than 0.2 second, which is very commonly used, as a result the covariance performed during the filtering process will differ from the sensors' inherent covariance. In this case, some good methods that determine parameter matrices of a filter based on the sensors' inherent noise levels will hardly work well. Secondly, noise levels of the attitude sensors are not immutable, but can be somewhat affected by the temperature, pressure, vibration and some other environmental conditions. In these cases, theoretical solutions for the actual covariance information become intractable, and it is much better to utilize the fuzzy concept. With the help of fuzzy logic, we can easily tune the filtering parameters to a better level.

* Corresponding authors.
E-mail addresses: wangxiaochu1985@gmail.com (X. Wang),
yz-dpi@mail.tsinghua.edu.cn (Z. You), kaichunz@mail.tsinghua.edu.cn (K. Zhao).
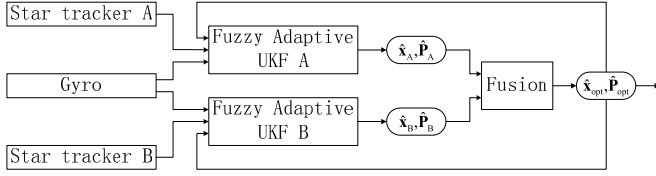
Fig. 1. Framework of the attitude determination algorithm.

This study deals with the attitude estimation algorithm of the satellite with an attitude measurement unit comprised of two star trackers and one gyroscope. Considering the practical problems mentioned above, a fuzzy adaptive algorithm is introduced into the standard UKF with minimal skew sampling method, outputting sub-optimal attitude states and gyro bias. And Covariance Intersection algorithm is then adopted to combine data between the two sub-systems, gaining the optimal system-level attitude estimation and gyro bias. Then the optimal states will be sent back to the sub-systems, taking part in the next filtering step. The algorithm framework in this work is shown in Fig. 1. The main contribution of this paper is threefold. Firstly, a modified attitude determination framework is proposed with feedback (from system level to sub-system level) and it is verified. Secondly, simple but effective compensation and fuzzy strategies are designed to solve some practical issues (i.e. asynchrony between observation units at the same time, and the resulting noises problem). Thirdly, a simple method for the fusion of non-additive parameters is proposed, targeting at the system level, which can avoid root square operation and can be directly extended to the fusion of multiple parameters, though this is not presented in this paper.

2. Representation and sensor model

2.1. Attitude representation

There are various attitude parameterizations for attitude representations, such as Euler angles, quaternions, and modified Rodrigues parameters. Among them, the quaternion is most widely used in the attitude representation of satellites, because it is a nonsingular representation and thus convenient for computer calculating. A quaternion representing attitude has a three-vector part and a scalar part:

$$\bar{q} = \begin{bmatrix} \boldsymbol{q} \\ q_4 \end{bmatrix} = \begin{bmatrix} \boldsymbol{e}\sin\frac{\phi}{2} \\ \cos\frac{\phi}{2} \end{bmatrix}. \tag{1}$$

The components of a quaternion obey the unit length constraint as:

$$|\bar{q}|^2 = |\boldsymbol{q}|^2 + q_4^2 = 1. \tag{2}$$

Attitude error can also be represented by modified Rodrigues parameters and rotation vectors, which are better than quaternions in small rotation conditions, because they can be calculated in the form of linearity instead of complex calculations with at least second-order precision. Details are discussed in the work of Ref. [9].

Euler's theorem states that the most general motion of a rigid body with one point fixed is a rotation by an angle ϕ about some axis, which we specify by a unit vector $\boldsymbol{e}$. Thus the rotation vector can be written as:

$$\boldsymbol{a}_\phi = \phi\boldsymbol{e} \tag{3}$$

and the modified Rodrigues parameters

$$\boldsymbol{p} \equiv \frac{\boldsymbol{q}}{1+q_4} = \boldsymbol{e}\tan\frac{\phi}{4} \equiv \frac{\boldsymbol{a}_p}{4}. \tag{4}$$

In small rotation conditions, $\boldsymbol{a}_p$ is approximately equal to angle ϕ as:

$$\boldsymbol{a}_p \approx \boldsymbol{\phi}. \tag{5}$$

Attitude error quaternion can be parameterized by the three-component representation of Eq. (4) as

$$\delta\bar{q}(\boldsymbol{a}_p) = \frac{1}{16+\boldsymbol{a}_p^2} \begin{bmatrix} 8\boldsymbol{a}_p \\ 16-\boldsymbol{a}_p^2 \end{bmatrix}. \tag{6}$$

Substitute Eqs. (4) and (5) into Eq. (6), noting that in the small rotation conditions the attitude error quaternion can be parameterized by the rotation angle, which is denoted by $\boldsymbol{a}$ for simplicity with the subscript ignored in this work, then we have:

$$\delta\bar{q}(\boldsymbol{a}) \approx \frac{1}{16+\phi^2} \begin{bmatrix} 8\phi\boldsymbol{e} \\ 16-\phi^2 \end{bmatrix} \approx \begin{bmatrix} \frac{\boldsymbol{a}}{2} \\ 1-\frac{a^2}{8} \end{bmatrix} \tag{7}$$

and in this work, the state vector is $\begin{bmatrix} \boldsymbol{q} \\ \boldsymbol{\beta} \end{bmatrix}$, error-state vector is $\begin{bmatrix} \boldsymbol{a} \\ \Delta\boldsymbol{\beta} \end{bmatrix}$.

2.2. Sensor model

MEMS gyros are usually used to measure the angular rates of satellites with respect to an inertial frame. Ref. [10] has demonstrated that MEMS gyros can be modeled by the same model as was proposed by Farrenkopf [11] as follows:

$$\boldsymbol{\omega}^{meas} = (\boldsymbol{\omega}_{B,I})^B + \boldsymbol{\beta} + \boldsymbol{\eta}_v \tag{8}$$

$$\dot{\boldsymbol{\beta}} = \boldsymbol{\eta}_u \tag{9}$$

where $\boldsymbol{\omega}^{meas}$ is the measured angular rate, $\boldsymbol{\omega}_{B,I}$ is the true angular rate with respect to the Earth Centered Inertial (ECI) frame, with subscript B denoting the body frame of the satellite, subscript I denoting the ECI frame respectively, and superscript B indicating that the quantity is observed in the body frame of the satellite. $\boldsymbol{\beta}$ is the drift of the gyro, and $\boldsymbol{\eta}_v, \boldsymbol{\eta}_u$ are two independent zero-mean Gaussian white noises, satisfying $\boldsymbol{\eta}_v \sim N(0, \sigma_v^2), \boldsymbol{\eta}_u \sim N(0, \sigma_u^2)$.

The basic measuring principle of the star tracker is that: the stars are so much far away from the earth that their positions can be treated as fixed in the ECI frame. So the basic measuring equation can be given as:

$$\boldsymbol{w}_i = \boldsymbol{T}_{S,I} \cdot \boldsymbol{v}_i \tag{10}$$

where $\boldsymbol{w}_i$ is the unit line of sight (LOS) vector from the earth to a certain star with respect to the reference frame of the star tracker, $\boldsymbol{v}_i$ is the unit LOS vector from earth to a certain star with respect to the ECI frame, and $\boldsymbol{T}_{S,I}$ is the transformation matrix from the ECI frame to the star tracker's reference frame. Vector $\boldsymbol{w}_i$ can be calculated using the focal length and the positions of the image points. After recognizing the star atlas, numbers of stars can be identified, and thus $\boldsymbol{v}_i$ can be also known. Then the determination of $\boldsymbol{T}_{S,I}$ is a typical Wahba's problem. Using QUEST algorithm or others, matrix $\boldsymbol{T}_{S,I}$ can be estimated, and then the star tracker will output the attitude information in the forms of Euler angles or quaternions. Note that the direct attitude output is not the satellite's attitude but the star tracker's, the setup matrix $\boldsymbol{T}_{S,B}$ should be given to obtain the attitude matrix of the satellite which is denoted by $\boldsymbol{A}_{B,I}$:

$$\boldsymbol{A}_{B,I} = \boldsymbol{T}_{S,B}^{\mathrm{T}} \cdot \boldsymbol{T}_{S,I}. \tag{11}$$

Rewriting Eq. (11) in the form of quaternions, one obtains Eq. (12) as:

$$\bar{q}_{B,I} = (\bar{q}_{S,B})^{-1} \otimes \bar{q}_{S,I} \tag{12}$$