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Observer-based vibration control of non-classical microcantilevers using extended Kalman filters

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ABSTRACT

In non-classical micro-beams, the strain energy of the system is determined by the non-classical continuum mechanics. In this study, we consider a closed-loop control methodology for suppressing the vibration of non-classical microscale Euler–Bernoulli beams with nonlinear electrostatic actuation. The non-dimensional form of the governing nonlinear partial differential equation of the system is introduced and converted into a set of ordinary differential equations using the Galerkin projection method. In addition, we prove the observability of the system and we design a state estimation system using the extended Kalman filter algorithm. The effectiveness and performance of the proposed control scheme and estimation system are demonstrated based on computer simulations. In addition, we compare the results with those obtained by the classical theory of microscale beams.

1. Introduction

At present, microelectromechanical system (MEMS)-based microcantilever beams have important roles in science and technology, e.g., in the atomic force microscopes used to inspect the surfaces of materials [1], microswitches applied to high frequency switching [2], and microresonators designed to vibrate at a specific frequency [3]. Their high efficiency, simple structure, and facile production of electrostatic forces make them one of the most common actuating and sensing methods used in MEMS systems. Microbeam structures with electrostatic actuation have many applications in industrial and scientific areas such as micropressure, mass sensors, microtate gyros, and microflexible joints [4,5].

Previous studies have derived the governing equations of motion for electrostatically actuated beams, while others have studied their vibration [6,7]. Pull-in instability has also been investigated widely, while several studies have focused on predicting the pull-in voltage and its properties [8–10].

The need to improve the performance and resolution of microscale instruments in existing high performance control systems has led many investigators to analyze vibration control for microbeams. In 1995, Cunningham designed a controller that suppresses the first two modes of vibration in a stainless steel microcantilever beam [11]. Nonlinear electrostatic actuators were employed by Wang to stabilize the motion of a vibrating clamped-free microbeam via a switching feedback control form [12]. To address unavoidable ground vibration in an effective manner, Yen et al. achieved active vibration control for a stroke scanning probe microscope using discrete sliding mode control [13]. In 2006, Zhang et al. proposed a rational linearizing feedback controller with a high gain observer to eliminate unwanted deflection in a microcantilever beam system [14],

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where the Rayleigh–Ritz method was used to reduce the order of the dynamical model of microcantilever beams. In most previous studies of the vibration control problem in electrostatically actuated microsystems, the governing equations of motion have been reduced by assuming the lumping of parameters. Using this simplification, Vagia designed a switching proportional-integral-derivative control algorithm in 2008 [15], while he proposed a sliding mode control technique in 2012 to handle the nonlinearity and uncertainty in the system parameters [16].

The characteristic beam thickness is of the order of microns and submicrons in many applications of MEMS-based microbeams. In recent decades, the size effect in microscale beams has been studied experimentally in some metals and polymers [17–19]. These experiments indicate that the classical strain-based mechanics theories cannot be used to explain the microstructure-dependent size effect. Thus, conventional continuum mechanics need to be developed using higher order or non-classical continuum theories to describe this size-dependent phenomenon at a small scale.

In 1994, Fleck and Hutchinson established a new type of higher-order or non-classical continuum theory called strain gradient theory [17]. Lam et al. developed this theory and introduced the modified strain gradient elasticity theory in 2003, which is one of the most successful and inclusive non-classical continuum theories [20]. Using this new theory, many researchers have derived the new governing equations of non-classical microbeams to predict their static and dynamic behaviors based on the new models obtained. Thus, we summarize some of these studies. In the following, when we refer to the strain gradient theory, we specifically mean the version of the theory obtained by Lam et al. [20].

In 2009, Kong et al. obtained the new governing equations of equilibrium for Euler–Bernoulli microbeams based on the strain gradient theory [21], where they also studied the static and dynamic behavior of this new model of microbeams [21]. In 2011, Kahrobaiyan et al. derived the nonlinear governing equation for Euler–Bernoulli microscale beams using a combination of the strain gradient theory and Hamilton's principle [22]. In another study, they investigated the free vibration of functionally graded strain gradient Euler–Bernoulli microbeams [23]. Later, the nonlinear static bending deformation, the post-buckling problem, and the nonlinear free vibration of non-classical Euler–Bernoulli microbeams were analyzed by Zhao et al. [24]. A microscale Timoshenko beam model was developed based on strain gradient theory by Wang et al. [25]. Ansari et al. considered the free vibration characteristics of microbeams made of functionally graded materials based on the strain gradient Timoshenko beam theory [26]. In 2013, the nonlinear forced vibration of non-classical strain gradient Euler–Bernoulli microbeams was considered by Vatankhah et al. [27]. In [28,29], the authors investigated the boundary stabilization and exact controllability of non-classical Euler–Bernoulli microbeams using a simple boundary control law, where the actuator dynamics model was not considered in the system and the boundary controller design was also obtained in the absence of any noise and disturbances.

In this study, we aim to achieve vibration control for non-classical strain gradient electrostatically actuated microcantilevers using the nonlinear control theory. The nonlinear model of the electrostatic actuator is considered in this system. Furthermore, the control targets are studied in the presence of measurements and process noise. To achieve this goal, we first determine the nondimensional form of the governing nonlinear partial differential equation (PDE) of an electrostatically actuated Euler–Bernoulli microbeam based on the strain gradient theory, before the Galerkin projection method is employed to reduce the order of the system and the state equations are presented in the modal space. A state observation system is also required to estimate the states needed in the control system. The estimation system is designed based on the extended Kalman filter (EKF). This system should estimate the states based on the lowest measurements to satisfy the microscale limits of implementation and fabrication.

2. System model formulation

The system considered is a clamped-free microbeam with uniform cross-section A, density ρ, length L, and thickness h. The governing PDE of motion and the corresponding boundary conditions, which are based on the strain gradient theory proposed by Lam et al. [20], are obtained using Hamilton's principle for a non-classical strain gradient Euler–Bernoulli microscale beam, as follows [21]:

$$\rho A \frac{\partial^2 w}{\partial t^2} + K_1 \frac{\partial^4 w}{\partial x^4} - K_2 \frac{\partial^6 w}{\partial x^6} = 0, \tag{1}$$

$$\begin{cases} w|_{(0,t)} = \frac{\partial w}{\partial x}\Big|_{(0,t)} = \frac{\partial^2 w}{\partial x^2}\Big|_{(0,t)} = 0, \\ \frac{\partial^3 w}{\partial x^3}\Big|_{(L,t)} = \frac{\partial^5 w}{\partial x^5}\Big|_{(L,t)} = 0, \\ K_2 \frac{\partial^4 w}{\partial x^4}\Big|_{(L,t)} - K_1 \frac{\partial^2 w}{\partial x^2}\Big|_{(L,t)} = 0, \end{cases} \tag{2}$$

where x and t represent the independent spatial and time variables, respectively, and $w(x,t)$ denotes the lateral deflection. Furthermore,

$$\begin{aligned} K_1 &= EI + \mu A\left(2l_0^2 + \frac{43}{225} l_1^2 + l_2^2\right), \\ K_2 &= \mu I\left(2l_0^2 + \frac{4}{5} l_1^2\right), \end{aligned} \tag{3}$$