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Adaptive Multi-stage Output Feedback NMPC using the Extended Kalman Filter for time varying uncertainties applied to a CSTR

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Abstract: Nonlinear Model Predictive control (NMPC) is one of the advanced control strategies for multi-dimensional nonlinear systems with constraints. With uncertainties present in the model, robust NMPC strategies are proposed in order to counteract the effects of the uncertainties and have a safe operation of the plant. Multi-stage NMPC offers a non-conservative alternative as it models the feedback information explicitly in the problem formulation by means of a scenario tree. In order to be robust to both the model uncertainties and the estimation error, we formulate a multi-stage output feedback NMPC strategy by creating additional scenarios by sampling the innovations and use observer equations to predict the future evolution of the plant. Since the observers such as the Extended Kalman Filter (EKF) can be used to estimate the uncertain parameters along with the states, the output feedback NMPC strategy is improved to be adaptive with respect to time varying uncertain parameters and the performance of the controller is improved. We demonstrate the advantages of the proposed adaptive scheme using a nonlinear Continuous Stirred Tank Reactor (CSTR) example.

Keywords: Model-based control, Output feedback nonlinear model predictive control, Adaptive NMPC, Extended Kalman Filter, Robust control

1. INTRODUCTION

As we progress in the 21st century, the industries are being highly regulated owing to environmental and safety considerations and as a result stringent constraints are being imposed on the processes. The critical challenge for the industries is then to increase the efficiency of the plant operation while all process specifications are met. Advanced control techniques are needed which can not only handle constraints but also improve the overall efficiency of the plant. Nonlinear Model Predictive control (NMPC) is one of the advanced techniques which can address economic objectives while satisfying the process constraints.

In NMPC, we define the control task as an optimization problem with a certain objective and a set of constraints. The objective can be a simple tracking term or it can be an economic objective. Economic objectives can be maximization of a product, minimization of time for the production, reducing the energy consumption and increasing the profit of the company etc. For the chosen objective and the given constraints, the behaviour of the plant is predicted for a given time period (known as prediction horizon) by solving the optimization problem and the sequence of optimal control inputs are obtained. The control input that was calculated for the first time step is then applied to the plant and the optimization procedure is repeated online at each sampling instant.

The performance of the controller depends on the accuracy of the prediction and thus on how well the model represents the reality. Different uncertainties can be part of a model in the form of plant disturbances, parametric uncertainty, unmodelled dynamics and errors in the initial conditions etc. The controller must be robust to all the uncertainties in order to satisfy the constraints and achieve an optimal operation of the plant. Multi-stage NMPC is one of the robust NMPC strategies to tackle the model uncertainties (see Lucia et al. (2013)). Min-max and tube based are other well known strategies (see Scokaert and Mayne (1998) and Mayne et al. (2011)). In multi-stage NMPC, the evolutions of the plant for each realization of the uncertainties are considered as different scenarios. In this approach, the feedback information is taken explicitly into account in the prediction. As the plant evolves in time, the control inputs in the future are calculated for the state of the plant at those instants when the measurement information is revealed. This fact is modelled in the problem formulation of the multi-stage NMPC by considering the future control inputs can be different from each other depending on the evolution of the different scenarios and thus can act as recourse to counteract the effects of the uncertainties. This makes the approach less-conservative compared to other approaches. If the scenario tree is an exact representation of the future uncertainties, multi-stage NMPC provides the optimal solution under the given feedback information structure by solving an open-loop optimization problem.

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The presence of estimation error poses the interesting challenge of not only predicting the evolution of the plant under uncertainty but also the evolution of the plant under the control actions that may happen in the future as a result of (future) estimation errors. Under the assumption that the system is observable and that the innovations sequence is bounded, an EKF based multi-stage NMPC strategy was proposed in Subramanian et al. (2014). In this paper, we aim at improving the performance of the controller by reducing the conservativeness of the approach further by including the information on estimated parameters also in the problem formulation. The parametric uncertainties are estimated online and adapted in the controller in a systematic way. At each time step, the EKF is used to estimate the state and the time varying uncertain parameters of the plant. This information is modelled in the scenario tree with the assumption that the parameter estimation error is bounded and that an upper bound for the change of parameters between the sampling times can be obtained.

2. STATE AND PARAMETER ESTIMATION USING THE EXTENDED KALMAN FILTER

The EKF is one of the widely used nonlinear filters for state estimation. It consists of predictor and corrector steps. In the predictor step, the nonlinear model of the system is used to predict the state estimate of the system from the known initial conditions (or the state estimate from the previous time instant) and the applied control input. In the corrector step, the predicted state estimate is then updated once the measurements are obtained. The nonlinear model of the plant is given by

$$x_k = f(x_{k-1}, u_{k-1}, d_{k-1}) + w_{k-1}, \tag{1}$$

$$y_k = h(x_k) + r_k. \tag{2}$$

Equation (1) describes the model of the plant, where $x_k \in \mathbb{R}^{n_x}$ is the state of the plant, $u_k \in \mathbb{R}^{n_u}$ is the control input, $w_k \in \mathbb{R}^{n_x}$ is the process noise, $d_k \in \mathbb{R}^{n_d}$ is the vector of uncertain parameters (possibly time varying) at a given time step k and $f : \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \times \mathbb{R}^{n_d} \rightarrow \mathbb{R}^{n_x}$ is the model of the plant. Equation (2) describes he measurement equation, where $y_k \in \mathbb{R}^{n_y}$ is the measured output, $r_k \in \mathbb{R}^{n_y}$ is the measurement noise and $h : \mathbb{R}^{n_x} \rightarrow \mathbb{R}^{n_y}$ is the measurement model. w_k and r_k are assumed to be white Gaussian noises with covariance matrices given by Q_k and R_k respectively. The dynamics of the uncertain parameters d_k can be given by

$$d_k = f_d(x_{k-1}, u_{k-1}, d_{k-1}), \tag{3}$$

where $f_d : \mathbb{R}^{n_x} \times \mathbb{R}^{n_u} \times \mathbb{R}^{n_d} \rightarrow \mathbb{R}^{n_d}$ is the model of the uncertain parameter. If the variation in the uncertain time varying parameter is slow, the dynamics of the parameters can be considered as $d_k = d_{k-1}$ and the Kalman update improves the estimation at each sampling time. In order to estimate the unknown parameters, the dynamics of the parameters are added to the model of the plant and the estimation scheme is applied to the augmented model. The augmented model is given by $x_k^{Aug} = f^{Aug}(x_{k-1}^{Aug}, u_{k-1}), f^{Aug} : \mathbb{R}^{n_x+n_d} \times \mathbb{R}^{n_u} \rightarrow \mathbb{R}^{n_x+n_d}$. The predictor step of the EKF is then given by,

$$\hat{x}_k^{Aug,-} = f^{Aug}(\hat{x}_{k-1}^{Aug}, u_{k-1}), \tag{4}$$

$$P_k^- = A_{k-1} P_{k-1} A_{k-1} + Q_k, \tag{5}$$

$$A_{k-1} = \left. \frac{\partial f^{Aug}}{\partial x} \right|_{\hat{x}_{k-1}^{Aug}}. \tag{6}$$

$\hat{x}_k^{Aug,-}$ consists of the predicted state and the parameter estimate. P_k is the covariance matrix of the estimation error. (5) gives the one-step ahead prediction of the covariance matrix. The measurement update is then performed via:

$$\hat{x}_k^{Aug} = \hat{x}_k^{Aug,-} + K_k(y_k - h(\hat{x}_k^{Aug,-}), \tag{7}$$

$$K_k = P_k^- C_k^T (C_k P_k^- C_k^T + R_k)^{-1}, \tag{8}$$

$$P_k = (I - K_k C_k) P_k^-, \tag{9}$$

where the Kalman gain K_k gives the update to the predicted estimate. The measurement equation is linearized and C_k is obtained by $C_k = \left. \frac{\partial h}{\partial x} \right|_{\hat{x}_k^{Aug}}$. $\nu_k = y_k - h(\hat{x}_k^{Aug,-})$ is the innovations which are the new information used to update the predicted estimate. (7) gives the state and the parameter estimate at the given time step k which are then used to initialize the control problem.

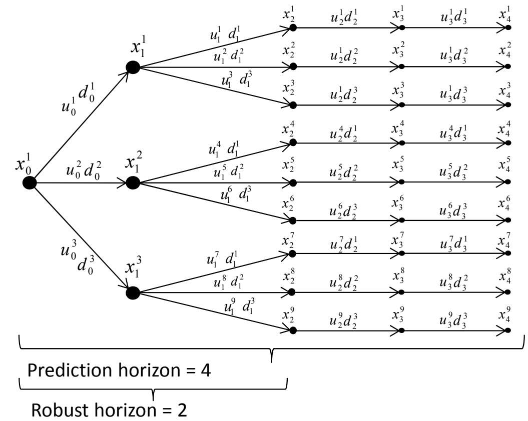

Fig. 1. Scenario tree representation of the uncertainty evolution for multi-stage NMPC.

3. MULTI-STAGE OUTPUT FEEDBACK NMPC

3.1 Multi-stage NMPC

The multi-stage NMPC is a non-conservative robust NMPC strategy (see Lucia et al. (2013)). In this method, the evolution of the system is represented using a scenario tree as shown in Fig. 1. The current state of the system forms the root node of the scenario tree. Each branch denotes the evolution of the plant for a possible realization of the uncertain parameters. The important aspect of the approach is that the feedback information is taken explicitly into account and that the decisions can be different for the scenarios branching from different nodes. This models the fact that the measurements are available at the future and the future inputs can act as recourse to counteract the effects of the uncertainties until that point in time. This makes the method less conservative than others. If the uncertainties are discrete valued, the problem formulation provides the optimal control policy. The stability properties of the method are discussed in Lucia et al. (2014b). Rigorous guarantees for the continuous valued uncertainties can be obtained by combining the method with reachability analysis as shown in Lucia et al. (2014c).

In the scenario tree, the decisions taken at every node must be the same because the future realizations are not known. The constraint which forces the inputs to be the same at a given node is called the non-anticipativity constraint (e.g. in Fig. 1, $u_0^1 = u_0^2 = u_0^2; u_1^1 = u_1^2 = u_1^3; \ldots$).