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Estimation of intra-operative brain shift based on constrained Kalman filter

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ABSTRACT

In this study, the problem of estimation of brain shift is addressed by which the accuracy of neuronavigation systems can be improved. To this end, the actual brain shift is considered as a Gaussian random vector with a known mean and an unknown covariance. Then, brain surface imaging is employed together with solutions of linear elastic model and the best estimation is found using constrained Kalman filter (CKF). Moreover, a recursive method (RCKF) is presented, the computational cost of which in the operating room is significantly lower than CKF, because it is not required to compute inverse of any large matrix. Finally, the theory is verified by the simulation results, which show the superiority of the proposed method as compared to one existing method.

1. Introduction

Image guided neurosurgery (IGNS) provides the exact position of surgical tools in the patient's body. This property is employed in computer-aided surgery and results in a more precise treatment. The accuracy of IGNS is largely dependent on the used images. Furthermore, due to the brain tissue flexibility, it deforms after dural opening; thus, pre-operative images are not valid anymore, which result in a less accurate navigation. This brain deformation is known as "brain shift" in which many factors are involved such as tumor resection, gravity, pharmacologic responses, and loss of cerebrospinal fluid. Another problem is that the precise impact of each of these factors is not specified [1,2]. There are mainly two solutions for the problem of brain shift: intra-operative medical imaging and biomechanical models of the brain. Intra-operative MRI (iMRI) [3], intra-operative CT (iCT) [4], and intra-operative Ultrasound (iUS) [5] fall into the first category. iMRI is time-consuming and requires expensive non-magnetic tools. iCT can be harmful to health in a long time due to high dose X-ray used in it. iUS is much cheaper than iMRI, but it generally results in lower quality images. The latter approach uses physical features of brain tissues that are expressed by partial differential equations (PDE). Afterwards, the PDE is solved using boundary conditions and the solution of which is used to improve pre-operative images. In addition, it results in high resolution images as a result of using pre-operative ones, on which this paper is also based.

Several models have been presented for assessing the biomechanical behaviour of brain tissues. These models consist of mass spring model [6], linear elastic model [7], nonlinear model [8], and mechanical model based on consolidation theory [9]. Mass–spring model is over-simplified and has low accuracy. In [10], it is shown that using an appropriate finite deformation solution, the choice of linear elastic and nonlinear models do not affect the accuracy. As a result, the linear elastic model is recommended due to its reduced computations. In [11], a comparison is performed between the mechanical model and linear elastic model that shows the elastic model is more accurate.

The measurement of brain surface displacement is employed in [12–17] with somewhat promising results. In [12], a method is presented to improve solution of linear elastic model using image processing theory, the main drawback of which is dependency on view angle. In [13], an atlas-based method is proposed and its sensitivity analysis is studied in [14]. Optimizations are also carried out based on calculus of variation [15], steepest gradient descent [16], and game theory [17]. The defect of the above-mentioned approaches is that only brain surface estimation error is minimized, but subsurface tissues estimation is not investigated.

One of the well-known approaches in estimation problems is Kalman filter which is utilized for parameter estimation of dual-

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rate systems [18] and MicroElectroMechanical systems [19]. If there are constraints on estimation problem, constrained Kalman filter (CKF) will result in a more accurate estimation [20,21], which is employed in [22–24]. In [22], state estimation problem of power systems is addressed. Two algorithms for estimation with inequality constraint are derived in [23], and a reduced-order Kalman filter is presented in [24].

In this paper, linear elastic model is adopted and the desired estimation is calculated using the CKF. To this end, brain shift is considered as a Gaussian random vector with a known mean and an unknown covariance. Then, the best estimation is found such that an upper bound of the estimation error variance will be minimized. Furthermore, a recursive method (RCKF) is presented to prevent computing inverse of a large matrix which results in a less computational cost in the operating room. Moreover, superiorities of the proposed approach over the existing methods is illustrated using finite element method. The claims are also demonstrated by simulation results.

2. Model and assumptions

According to the Introduction, the linear elastic model is utilized as governing equations of the brain in this paper. This model with corresponding assumptions for estimation are explained in this section.

2.1. Linear elastic model

The linear elastic model considers brain as a linear elastic continuum with no initial stresses or strains. The energy of brain deformation due to the external forces can be expressed as [25]

$$E=\frac{1}{2}\int_{\Omega}\sigma^T\varepsilon\, d\Omega+\int_{\Omega}F^T u\, d\Omega \tag{1}$$

where F is the total external force applied to the body, Ω is the elastic body, u is the displacement vector, ε is the stress vector, and σ is the strain vector. The relationships between stress, strain and displacement vector are as follows:

$$\sigma=D\varepsilon,$$
$$\varepsilon=Lu$$

where L is an operational matrix and D is the elasticity matrix that describes properties of the body [25]. According to the *principle of minimum potential energy* [26], only the actual value of u can minimize (1) for a certain body. Hence, any estimation of u is the suboptimal solution of minimizing (1). If finite element method (FEM) be used to minimize (1), volume of the brain will be discretized and yields the following equation for brain model [27]:

$$Kx=b \tag{2}$$

where $b\in\mathbb{R}^n$ includes boundary conditions and surface forces, $x\in\mathbb{R}^n$ expresses discrete quantities of u in the FEM's nodes, and $K\in\mathbb{R}^{n\times n}$ is the stiffness matrix that has the role of discrete equations. Unfortunately, measurement of b is not possible in the operation room and consequently it is not possible to calculate x from (2) directly. As mentioned in the Introduction, brain surface imaging can be utilized to improve brain shift estimation. Since x expresses position of all nodes in the brain, a full row rank matrix C can be found such that the position of brain surface is calculated from x. Then it can be assumed that

$$y=Cx \tag{3}$$

where $y\in\mathbb{R}^m$ $(m<n)$ is the measurement of brain surface. Therefore, it is not possible to calculate x from y directly.

2.2. Assumptions

The following assumptions are made to solve the problem:

(i) $\mathrm{rank}(K)=n$.
(ii) An initial estimation $\overline{b}$ of b is available.

It is noted that none of the assumptions is restricting. The first assumption held due to (2) is obtained from a valid FEM and proper boundary conditions [28]. The second assumption states that an initial estimation of b is available, and to this end physics of the problem can be used. For instance, it can be assumed that $\overline{b}$ is due to the gravity and loss of cerebrospinal fluid [29].

3. Brain shift estimation using constrained Kalman filter

To estimate brain shift by CKF, the vector b is considered as a Gaussian random vector with mean vector $\overline{b}$ and unknown covariance matrix. Therefore, x is also a Gaussian random vector with unknown covariance matrix, and its mean vector $(\overline{x})$ using (2) can be given by

$$\overline{x}=K^{-1}\overline{b}. \tag{4}$$

One way to compute an estimation of x $(\hat{x})$ is to estimate b, then $\hat{x}$ can be computed from (2) as

$$\hat{x}=K^{-1}\hat{b} \tag{5}$$

where $\hat{x}$ and $\hat{b}$ are estimates of x and b, respectively. This approach is known as *inverse method* [29,16]. The purpose of estimation $\hat{b}$ is to minimize variance of estimation error of b; therefore, the following cost function is considered:

$$J=\mathrm{E}\left[(b-\hat{b})^T(b-\hat{b})\right]. \tag{6}$$

If the cost function is minimized without any constraint, $\hat{b}$ will be found identical to $\overline{b}$. To improve the estimation, it is needed to define a well-suitable constraint. By substitution of (2) into (3), one can get

$$y=CK^{-1}b.$$

It is obvious that the estimation of b should satisfy this equation. Consequently, the following constraint can be considered:

$$y=CK^{-1}\hat{b}. \tag{7}$$

Now $\hat{b}$ should minimize the cost function (6) subject to the constraint (7). To solve the problem, the constraint can be adjoined to the cost function using Lagrange multipliers; therefore, the resulting augmented cost function can be given as

$$J_a=\mathrm{E}\left[(b-\hat{b})^T(b-\hat{b})\right]+2\lambda^T\left(y-CK^{-1}\hat{b}\right).$$

The expanded form of J_a is

$$J_a=\int b^T b f(b)\, db-2\hat{b}^T\int b f(b)\, db+\hat{b}^T\hat{b}\int f(b)\, db+2\lambda^T\left(y-CK^{-1}\hat{b}\right).$$

The integral of the second term is the mean of b, i.e. $\overline{b}$. Moreover, by considering the properties of probability density function, the integral of the third term is equal to one; therefore, the final equation can be expressed as

$$J_a=\int b^T b f(b)\, db-2\hat{b}^T\overline{b}+\hat{b}^T\hat{b}+2\lambda^T\left(y-CK^{-1}\hat{b}\right). \tag{8}$$

To minimize (8), the following equations must hold:

$$\frac{\partial J_a}{\partial\hat{b}}=-2\overline{b}+2\hat{b}-2K^{-T}C^T\lambda=0, \tag{9}$$