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Two Modified Unscented Kalman Filter and Acceleration Information in Unmanned Surface Vehicle Estimation

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Abstract: The stability problem is one of the existing problems of unscented Kalman filter (UKF), which due to the nonlinearity and complexity of system. The precision will be decreased or even UKF will be halted when the algorithm can't ensure the state covariance to be positive semidefinite. Based on decomposition of state covariance, the paper first reviews two modified UKFs which can enhance the state covariance to be positive semidefinite, then describes a method which combines the modified UKFs with acceleration measurement together to unmanned surface vehicle (USV) state estimation which main advantage lies in binding UKF with acceleration to simplify the system estimation model, so that it becomes an online estimation algorithm with higher precision and lower calculation complexity.

Keywords: diagonal similar decomposition, acceleration, singular value decomposition, positive semidefinite.

1. INTRODUCTION

Online estimation aims mainly to obtain high precise states or parameters of a dynamic system with a set of noisy measured signals. Kalman filter (KF) provides acceptable solutions in many situations(Li Ju, Zhao Fang,1989; Ji Liangzhao,1985; Fang Jiancheng, Shen GongXun,1998). However, the evolution of systems is governed by nonlinear functions in a larger of real applications, such as robots, industrial process, and even economy systems, and this makes the validity of KF to drop greatly(Youmin Zhang, Guanzhong Dai, Hongcai Zhang,1995). Thus, nonlinear estimation now becomes an interesting and growing research area.

In the past decades, many approximate optimal nonlinear estimation methods, including the extended Kalman filter (EKF) and the unscented Kalman filter (UKF), have been developed and applied to all kinds of practical or simulation nonlinear systems. EKF, which generalizes KF method to nonlinear systems is most commonly used in real applications, but it also exhibits several severe disadvantages, for example heavy computational burden due to the Jacobian Matrix computation and poor estimation precision for systems with high nonlinearities. Researchers proposed a new KF based nonlinear estimation strategy – UKF(R. E. Kalman,1960; S. J. Julier, J. K. Uhlmann， H. F. Durrant-Whyte,1995; E. A. Wan, R. Van der Merwe,2000) to improve the performance of EKF method. It has been shown that UKF can approximate the posterior mean and covariance with a second order accuracy(K. Xiong, H.Y. Zhanga, C.W. Chanb,2006) when mean and covariance is the Gaussian random variable. This makes UKF has higher precision than EKF algorithm. Many researches on the performance of UKF can be found in recent years and can be divided into two kinds: 1) algorithm improvement and 2) applications.

References(Y. Q. Chen, H. Thomas, Y. Rui,2002; C. Song, H. Sharif, K. Nuli,2004; M. Yamakita, Y. Musha, G. Kinoshita,2004; Zhexue Ge, Yongmin Yang, Xingwei Wang, Zheng Hu,2005; M. C. VanDyke, J. L. Schwartz, C. D. Hall,2004) present the applications of UKF algorithm in many kinds of fields. Such as, based on nonlinear observation and nonlinear kinetics model Chen et al. (Y. Q. Chen, H. Thomas, Y. Rui,2002) studied UKF parameters estimation; Yamakita et al.(M. Yamakita, Y. Musha, G. Kinoshita,2004) compared the recursive least squares, adaptive observer, EKF and UKF and got a conclusion that the smaller error and the UKF algorithm had smaller error and higher precision; Ge Zhe-Xue et al. (Zhexue Ge, Yongmin Yang, Xingwei Wang, Zheng Hu,2005)put forward a fault detect method based on UKF; UKF has been also introduced to the aircrafts modeling (M. C. VanDyke, J. L. Schwartz, C. D. Hall,2004).

Besides the application researches of UKF, many researchers also try to improve the performance of nominal UKF algorithm and many new version UKF algorithms are put forward. Such as, in 2001, square root UKF (R. Van der Merwe, E. A. Wan,2001) was proved that the numeric stability and parameter estimation efficiency was higher (for special situation of parameter estimation is $O(L^2)$) than nominal UKF by Van der Merwe and Wan. The revised UKF, which could reach Cramér–Rao lower bound (CRLB) with a certain conditions, is proposed by K. Xiong(K. Xiong, H.Y. Zhanga, C.W. Chanb,2006). Song Qi (Qi Song , Zhe Jiang, Jianda Han,2007) could estimate the covariance of noise online with the proposed Adaptive UKF (AUKF).

UKF has been the focus of research of estimation online. When the state covariance matrix is not positive semidefinition, the estimation of states and outputs maybe complex, which is not consistent with the measurement. In this paper, two modified UKFs--singular value decomposition UKF (SVDUKF)(Yulong Ma, Zhiqian Wang, Yuqing He, Jianda Han, Xingang Zhao, 2010) and diagonal similar decomposition UKF (DSDUKF) (MA Yu-Long, ZHAO Xin-gang, WANG Zhi-Qian, HAN Jian-Da,2009),

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which are based on state covariance matrix decomposition, are proposed to ensure the positive semi definition of the state covariance matrix, then a method, which combines the modified UKFs with acceleration measurement together is put foreword to unmanned surface vehicle (USV) motion state estimation, which main advantage lies in binding the ability of UKF to deal the strong nonlinearity with the feather of acceleration with much disturbance information to simplify the system estimation model, so that it becomes an online estimation algorithm with higher precision and lower calculation complexity.

2. MODIFIED UKF ALGORITHM

Consider the following nonlinear system with addictive noises,

$$\begin{cases} x_k = f(x_{k-1}, u_k) + v_k \\ y_k = Hx_k + w_k \end{cases} \quad (1)$$

where, $x_k \in R^n$ is the states vector, $u_k \in R^r$ and $y_k \in R^p$ are the input and output vectors at time instant k, respectively. v_k and w_k are the process noise and measurement noise such that

$E[v_k] = 0, E[w_k] = 0,$

$E[w_k w_j^T] = R_k \delta_{kj},$

$E[v_k v_j^T] = Q_k \delta_{kj}$

Here, we suppose that v_k, w_k, x_k are not correlative to each other.

Nominal UKF algorithm for the associated noisy nonlinear system is described as follow,

$$\begin{cases} \overline{x}_0 = E[x_0] \\ P_0 = E\left[(x_0 - \overline{x}_0)(x_0 - \overline{x}_0)^T\right] \end{cases} \quad (2)$$

$$\begin{cases} w_0^m = \dfrac{\lambda}{n+\lambda} \\ w_0^c = \dfrac{\lambda}{n+\lambda} + (1-\alpha^2+\beta) \\ w_i^m = w_i^c = \dfrac{1}{2(n+\lambda)} \qquad i = 1, \ldots, 2n \end{cases} \quad (3)$$

$$\chi_{k-1} = \left[\overline{x}_{k-1}, \overline{x}_{k-1} + \sqrt{(n+\lambda)P_{k-1}}, \overline{x}_{k-1} - \sqrt{(n+\lambda)P_{k-1}}\right] \quad (4)$$

$$\begin{cases} \chi_{k|k-1}^* = f(\chi_{k-1}, u_k) \\ \hat{x}_{k|k-1} = \sum_{i=0}^{2n} w_i^m \chi_{i,k|k-1}^* \\ P_{k|k-1} = \sum_{i=0}^{2n} w_i^c (\chi_{i,k|k-1}^* - \hat{x}_{k|k-1})(\chi_{i,k|k-1}^* - \hat{x}_{k|k-1})^T + Q \\ \gamma_{k|k-1} = h(\chi_{k|k-1}^*) \\ \hat{y}_{k|k-1} = \sum_{i=0}^{2n} w_i^m \gamma_{i,k|k-1} \end{cases} \quad (5)$$

$$\begin{cases} P_{\overline{y}_k \overline{y}_k} = \sum_{i=0}^{2n} w_i^c (\gamma_{i,k|k-1} - \overline{y}_{k|k-1}) \cdot (\gamma_{i,k|k-1} - \overline{y}_{k|k-1})^T + Q^n \\ P_{\overline{x}_k \overline{y}_k} = \sum_{i=0}^{2n} w_i^c (\chi_{i,k|k-1} - \overline{x}_{k|k-1}) \cdot (\gamma_{i,k|k-1} - \overline{y}_{k|k-1})^T \\ K_k = P_{\overline{x}_k \overline{y}_k} P_{\overline{y}_k \overline{y}_k}^{-1} \\ P_k = P_{k|k-1} - K_k P_{\overline{y}_k \overline{y}_k} K_k^T \\ \overline{x}_k = \overline{x}_{k|k-1} + K_k (y_k - \overline{y}_{k|k-1}) \end{cases} \quad (6)$$

where $\lambda = (N+\kappa)\alpha^2 - N$, α is the constant to control the *Sigma point* distribution, κ is a scaling parameter which is usually set to 0, and β is nonnegative constant.

The computational cost of the nominal UKF algorithm is similar as the EKF, but computational precision is much higher. Furthermore the Jacobian Matrix of the system function needs not to be computed any more.

Next let us interpret the modified UKF. If the covariance of the state is not positive semidefinite, we can force it to be. For example the singular value decomposition of state covariance can do. Replacing the equation (4) of the UKF algorithm with equation (7) (Yulong Ma, Zhiqian Wang, Yuqing He, Jianda Han, Xingang Zhao, 2010) or equation (8) (MA Yu-Long, ZHAO Xin-gang, WANG Zhi-Qian, HAN Jian-Da, 2009), the new measurement update in algorithm are

$$\begin{cases} [U, D, V] = f_{\text{svd}}(P_{k-1}) \\ P_M = U\left(\sqrt{(n+\lambda)D}\right)U^{\mathrm{T}} \\ \chi_{k-1} = [\hat{x}_{k-1}, \hat{x}_{k-1} - P_M, \hat{x}_{k-1} + P_M] \end{cases} \quad (7)$$

where $D = \text{diag}(\sigma^2_1, \sigma^2_2, \sigma^2_3, \cdots \sigma^2_n)$, $\sigma_i (i = 1, 2, \cdots n)$ are the singular values of P (including 0). $f_{\text{svd}}(\bullet)$ is the function to get the eigenvectors and singular values of the matrix (•).

$$\begin{cases} [V_\lambda, D_\lambda] = eig(P_{k-1}) \\ P_m = V_\lambda\left(\sqrt{(n+\lambda)|D_\lambda|}\right)V^T \\ \chi_{k-1} = [\overline{x}_{k-1}, \overline{x}_{k-1} + P_m, \overline{x}_{k-1} - P_m] \end{cases} \quad (8)$$

where $D_\lambda = diag(\lambda_1, \lambda_2, \lambda_3, \cdots \lambda_n)$, $\lambda_i (i = 1, 2, \cdots n)$, $eig(\bullet)$ is the function to get the eigenvectors and eigenvalues of the matrix (•).

3. ACCELERATION ENHANCED SVDUKF TO USV

3.1 USV system model

Ignoring the swing, pitch and roll, assuming inertia matrix, added mass and damping matrix are diagonal matrixes, considering the interference of wind, wave, flow, motion model surface vessels usually simplified to non-planar three degrees of nonlinear equations as follow(K.D. Do, Z.P. Jiang, J. Pan,2004):