A Hybrid Current-Power Optimal Power Flow Technique

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***Abstract*—An equivalent current injection (ECI)-based hybrid current-power optimal power flow (OPF) model is proposed in this paper, and the predictor-corrector interior point algorithm (PCIPA) is tailored to fit the OPF for solving nonlinear programming (NLP) problems. The proposed method can further decompose into two subproblems. The computational results of IEEE 9 to 300 buses have shown that the proposed algorithms can enhance the performance in terms of the number of iterations, memory storages, and CPU times.**

***Index Terms*—Equivalent current injection, nonlinear programming, optimal power flow, predictor-corrector interior point algorithm.**

I. Introduction

OPTIMAL power flow was first discussed [1] in 1962 and took a long time to become a successful algorithm that could be applied for everyday uses [2], [3]. OPF can be applied not only in the system planning but also in the real-time operation for power systems in the deregulation environment. Reference [4] provided an overall introduction on the lambda-iteration method, gradient method, Newton's method, and the linear programming (LP) technique for solving OPF problems.

With Karmarkar's publication [5] in 1984, many interior point algorithms (IPAs) for the linear programming and quadratic programming (QP) have been proposed. In recent years, the primal-dual interior point algorithm (PDIPA) has been extensively applied to solve problems such as the OPF [6], [7], state estimation [8], security constrained OPF [9], and optimal reactive power flow [10]. Numerical results show that PDIPA has a great potential for solving problems of power systems operation and planning, as compared with many conventional methods, including the Newton's method [11].

In 1992, Mehrotra proposed best-search directions that defined the predictor and corrector steps which then generated the PCIPA [12]. The use of the PCIPA may improve the convergent performance, resulting in a small number of iterations.

A current injection algorithm based on the use of a constant nodal admittance matrix was described in [13], which discussed, in a tutorial nature, that this algorithm cannot be used for general power flow (PF) applications because a satisfactory method of modeling generator PV nodes with currents has not yet been developed, which could cause convergent instability or even divergence.

Experiencing these PV difficulties in publishing [14], [15] by the author(s), current based power flow of [14] was developed for distribution networks *only*, where generator PV buses are not common and can be omitted. We can get a constant Jacobian matrix which needs to be factorized only once. Reference [15] successfully implements the current power flow for high voltage networks, with a new idea of resolving the PV bus by using a single active power mismatch equation and an associated voltage deviation instead of the intuitive current conversion which could cause divergence. We can get a nearly constant Jacobi with a few generator buses still state-dependent and need to be updated at each iteration.

Pioneering the rectangular-form current-based OPF, [16] did a brief test with rectangular nodal voltages and branch currents used for state variables. The generator PV problem was avoided by replacing the PV bus with real and reactive power (PQ) directly; however, the oversimplification by replacing PV with PQ is not a common practice in handling generator buses. Besides, using KCL in [16], it was not even mentioned how load and generator power injections are handled for each iteration, which are the key factors affecting convergent behaviors in developing a current-based model. Reference [17] developed a rectangular voltage OPF, but the power flow equations are still PQ based, not current.

The constrained nonlinear optimization problem in this paper is solved using PCIPA that permits the efficient and effective handling of large sets of equality (power flow) and inequality (limits) constraints. The OPF uses rectangular form for both the voltage and current, and current mismatch equations are used for power flow calculation with the PV buses specifically treated by the model of [15] to ensure the numerical stability. The OPF problem can also be decoupled into two small subproblems [18] to further enhance the performance. Optimization can be accomplished by repeatedly solving the two subproblems.

II. Notation

The following symbols are used throughout this paper. Some symbols are also defined in the text where they first appear.

Symbols

$\Delta(\cdot)$	Change in variables.
$\nabla(\cdot)$	Differentiation operation.
$\underline{(\cdot)}, \overline{(\cdot)}$	Subscripts denoting lower and upper limit.

Manuscript received November 29, 2006; revised August 30, 2007. Paper no. TPWRS-00831-2006.

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Digital Object Identifier 10.1109/TPWRS.2007.913301

$(\cdot)^T$	Superscript denoting transpose.
$(\cdot)^{spec}$	Specified constant.
$(\cdot)^{cal}$	Calculated value of each iteration.
P	Active power.
Q	Reactive power.

Vectors

$\underline{\omega}$	Lower limit slack variables for inequality constraints.
$\overline{\omega}$	Upper limit slack variables for inequality constraints.
$\underline{z}$	Lower limit dual variables for inequality constraints.
$\overline{z}$	Upper limit dual variables for inequality constraints.
$\underline{h}$	Inequality constraints lower limit.
$\overline{h}$	Inequality constraints upper limit.
λ	Lagrangian multiplier for power flows.
x	Problem variables for minimum cost.
$\tilde{e}$	Column vector of ones.

Matrices

Y_G	Real component of Y (admittance) matrix.
Y_B	Imaginary component of Y matrix.
H	Augmented Hessian matrix.
$\underline{W}$	Diagonal matrix: $diag(\underline{\omega}_j)$.
$\overline{W}$	Diagonal matrix: $diag(\overline{\omega}_j)$.
$\underline{Z}$	Diagonal matrix: $diag(\underline{z}_j)$.
$\overline{Z}$	Diagonal matrix: $diag(\overline{z}_j)$.

III. Equivalent Current Injection Model

The complex bus voltages are defined in Cartesian form as

$$V_i = e_i + jf_i \tag{1}$$

where e_i and f_i are, respectively, the real and imaginary components of V_i.

A. Equations for PQ Buses

From the transmission line π model in Fig. 1, the rectangular form current injections are

$$\begin{aligned} I_i = &\{g_{ij}(e_i - e_j) - b_{ij}(f_i - f_j) - b_c f_i\} \\ &+ j\{g_{ij}(f_i - f_j) + b_{ij}(e_i - e_j) + b_c e_i\} \end{aligned} \tag{2}$$

$$\begin{aligned} I_j = &\{g_{ij}(e_j - e_i) - b_{ij}(f_j - f_i) - b_c f_j\} \\ &+ j\{b_{ij}(e_j - e_i) + g_{ij}(f_j - f_i) + b_c e_j\} \end{aligned} \tag{3}$$

where $I_i = I_{i,r} + I_{i,i}$ and $I_j = I_{j,r} + I_{j,i}$.

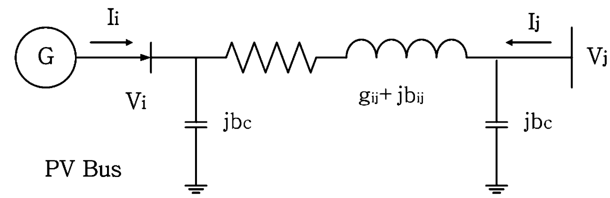

Fig. 1. Transmission line equivalent π model.

From above, the Newton–Raphson algorithm can be written in the ECI form [14] at the kth iteration by considering all PQ buses, that is

$$\left[\begin{array}{c} \Delta I_{i,r}^k \\ \hline \Delta I_{i,i}^k \end{array}\right] = \left[\begin{array}{c|c} Y_G & -Y_B \\ \hline Y_B & Y_G \end{array}\right] \left[\begin{array}{c} \Delta e_i^k \\ \hline \Delta f_i^k \end{array}\right] \tag{4}$$

where the current mismatches are defined by the specified value (spec) minus the calculated (cal) value as

$$\begin{aligned} \Delta I_{i,r} &= \Delta I_{i,r}^{spec} - \Delta I_{i,r}^{cal} \\ \Delta I_{i,i} &= \Delta I_{i,i}^{spec} - \Delta I_{i,i}^{cal} \end{aligned} \tag{5}$$

and

$$\begin{aligned} e^{k+1} &= e^k + \Delta e^k \\ f^{k+1} &= f^k + \Delta f^k. \end{aligned} \tag{6}$$

The specified constant power load P^{spec} and Q^{spec} can be converted into the specified ECI current load [14] with the calculated voltage for bus i at the kth iteration by

$$I_i^{spec} = \frac{(P_i - jQ_i)^{spec}}{\left(V_i^k\right)^*} = I_{i,r}^{spec} + jI_{i,i}^{spec}. \tag{7}$$

B. Representation of PV Buses

For a PV bus, its injected real power and voltage are given by

$$P_i = \mathrm{Re}\,[V_i \times I_i^*] = e_i \cdot I_{i,r} + f_i \cdot I_{i,i} \tag{8}$$

$$|V_i|^2 = e_i^2 + f_i^2. \tag{9}$$

Using Taylor's expansion of (8) and (9) [15] to substitute for ΔI in (4), it can get

$$\begin{aligned} &\left[\begin{array}{c} \Delta P_i \\ \hline \Delta |V_i|^2 \end{array}\right] = \left[\begin{array}{c|c} J_1 & -J_2 \\ \hline J_3 & J_4 \end{array}\right] \left[\begin{array}{c} \Delta e_i \\ \Delta e_j \\ \hline \Delta f_i \\ \Delta f_j \end{array}\right] \\ &J_1 = \left[\left(e_i \cdot g_{ij} + f_i \cdot b'_{ij} + I_{i,r}\right) \ \left(-e_i \cdot g_{ij} - f_i \cdot b_{ij}\right)\right] \\ &J_2 = \left[\left(-e_i \cdot b'_{ij} + f_i \cdot g_{ij} + I_{i,i}\right) \ \left(e_i \cdot b_{ij} - f_i \cdot g_{ij}\right)\right] \\ &J_3 = [2e_i 0] \\ &J_4 = [2f_i 0] \end{aligned} \tag{10}$$