Generalized Dynamic VSC MTDC Model for Power System Stability Studies

Stijn Cole, *Student Member, IEEE*, Jef Beerten, *Student Member, IEEE*, and Ronnie Belmans, *Fellow, IEEE*

***Abstract*—In this paper, a new general voltage source converter high voltage direct current (VSC MTDC) model is derived mathematically. The full system model consists of the converter and its controllers, DC circuit equations, and coupling equations. The main contribution of the new model is its valid for every possible topology of the DC circuit. Practical implementation of the model in power system stability software is discussed in detail. The generalized DC equations can all be expressed in terms of matrices that are byproducts of the construction of the DC bus admittance matrix. Initialization, switching actions resulting in different topologies and simulation of the loss of DC lines amount to a simple calculation or recalculation of the DC bus admittance matrix. The model is implemented in Matlab. Examples on a two- and six-terminal system show that the new model is indeed capable of accurately simulating VSC MTDC systems with arbitrary topology.**

***Index Terms*—HVDC converters, HVDC transmission, power system modeling, power system simulation.**

I. Introduction

With an ever increasing number of installations, voltage source converter high voltage direct current (VSC HVDC) systems become more and more important in the power system world. One advantage of VSC HVDC compared to conventional current source converter (CSC) HVDC is that the extension to multi-terminal DC (MTDC) systems is relatively easy. Although up until now, there are no VSC MTDC systems installed or even in concrete planning phase, quite a number of publications are devoted to the subject of VSC MTDC. They deal with various aspects of MTDC operation such as location of DC faults and protection [1], [2], control strategy [3], harmonics [4], and applications in wind farms [5], [6]. Also, numerous papers exist on the modeling of two-terminal VSC HVDC systems [7]–[10] and some on the modeling of multi-terminal CSC HVDC systems and their implementation in stability programs, e.g., [11] and [12]. As for basic modeling of VSC MTDC systems in power system stability programs, literature is limited to nonexistent. The multi-terminal models that are used in the aforementioned papers on MTDC have a fixed topology. It is not explained how to extend the models to other topologies. In case of CSC HVDC, this would be understandable because topologies are limited to a few simple configurations anyhow, due to the inherent difficulties associated with extending CSCs to multi-terminal systems. Furthermore, there already exist two multi-terminal CSC HVDC systems. Research is mostly limited to the particular topologies of those existing systems. For VSCs the situation is quite different. Firstly, as we already mentioned, they can be easily extended to multi-terminal systems, possibly with a large number of converters. Secondly, it is hard to predict which topology future MTDC systems will have. While it can be conjectured that the first actual VSC MTDC system will have a simple topology with a limited number of converters, quite a wide variety of "supergrid" topologies are conceivable.

This paper aims to redress this lacuna by proposing a general dynamic model of VSC MTDC, valid for *every conceivable topology of the DC circuit*. The model allows adding or removing converters and lines without changing the model itself. The model is susceptible of integration with large step size, so that it can be used for the simulation of large scale AC/DC systems. A converter model including controls is first derived in Section II. In Section III, the general DC circuit equations are derived. It is explained in Section IV how to combine the converter and DC circuit equations to represent any VSC MTDC system. In Section V, practical implementation of the general MTDC equations in transient stability programs is discussed. The paper is concluded with simulations that show the generality of the model.

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S. Cole and J. Beerten are with the Department of Electrical Engineering, K.U.Leuven, Leuven 3001, Belgium (e-mail: stijn.cole@esat.kuleuven.be; jef.beerten@esat.kuleuven.be).

R. Belmans is with the Department of Electrical Engineering, K.U.Leuven, Leuven 3001, Belgium, and also with Elia, Brussels, Belgium (e-mail: ronnie.belmans@esat.kuleuven.be).

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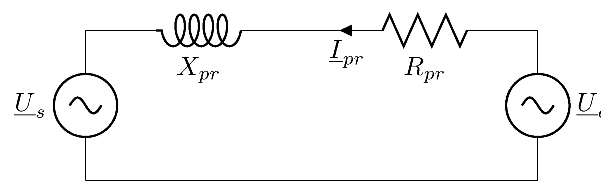

Fig. 1. One-line diagram of the AC circuit. Subscript s for system side. Subscript c for converter side.

II. Converter Modeling

VSC HVDC converters are connected to the system through a phase reactor, represented here by an impedance $Z_{\rm pr} = R_{\rm pr} + jX_{\rm pr}$, that can also include the effect of the transformer (Fig. 1). The basic equation of this circuit

$$u_c - u_s = L_{\rm pr}\frac{di_{\rm pr}}{dt} + R_{\rm pr}i_{\rm pr} \tag{1}$$

can be transformed to a power system synchronized rotating *dq* reference frame

$$\frac{d\left(i_{\mathrm{pr}}^{d}e^{j\omega t}\right)}{dt} = -\frac{R_{\mathrm{pr}}}{L_{\mathrm{pr}}}i_{\mathrm{pr}}^{d}e^{j\omega t} + \omega i_{\mathrm{pr}}^{q}e^{j\omega t} + \frac{1}{L_{\mathrm{pr}}}\left(u_c^d - u_s^d\right)e^{j\omega t} \tag{2a}$$

$$\frac{d\left(i_{\mathrm{pr}}^{q}e^{j\omega t}\right)}{dt} = \frac{R_{\mathrm{pr}}}{L_{\mathrm{pr}}}i_{\mathrm{pr}}^{q}e^{j\omega t} - \omega i_{\mathrm{pr}}^{d}e^{j\omega t} + \frac{1}{L_{\mathrm{pr}}}\left(u_c^q - u_s^q\right)e^{j\omega t}. \tag{2b}$$

The angle ωt is provided by the phase-locked loop (PLL), arbitrarily assumed here to align system voltage with the q-axis. Dividing by $e^{j\omega t}$, the dynamic equations of the AC circuit in the *dq*-frame are finally obtained:

$$\frac{di_{\mathrm{pr}}^{d}}{dt} = -\frac{R_{\mathrm{pr}}}{L_{\mathrm{pr}}}i_{\mathrm{pr}}^{d} + \omega i_{\mathrm{pr}}^{q} + \frac{1}{L_{\mathrm{pr}}}\left(u_c^d - u_s^d\right) \tag{3a}$$

$$\frac{di_{\mathrm{pr}}^{q}}{dt} = -\frac{R_{\mathrm{pr}}}{L_{\mathrm{pr}}}i_{\mathrm{pr}}^{q} - \omega i_{\mathrm{pr}}^{d} + \frac{1}{L_{\mathrm{pr}}}\left(u_c^q - u_s^q\right). \tag{3b}$$

Most VSC converters have a cascaded control structure, comprising inner current control loop and outer controllers [13]. The controllers themselves are usually PI controllers, “the de facto industry standard for current control”, that guarantee high performance [14], and also used in, e.g., [15]. In this paper, the emphasis is not placed on the control scheme. Therefore, a standard control scheme, consisting of cascaded PI controllers, is implemented here.

A. Current Controllers

The converter (3a) and (3b), obtained in the previous section, can be controlled by two parallel control loops, using PI controllers. The cross-coupling terms

$$\Delta u_c^d = \omega L_{\mathrm{pr}} i_{\mathrm{pr}}^{q} \tag{4}$$

$$\Delta u_c^q = -\omega L_{\mathrm{pr}} i_{\mathrm{pr}}^{d} \tag{5}$$

compensate for the cross coupling between the two control loops, introduced in (3a) and (3b) by the transformation to a rotating reference frame. The reference voltage in d and q components are then expressed as

$$u_{c^{\mathrm{ref}}}^{d} = u_s^d + \Delta u_c^d + \left(K_{p1} + \frac{K_{i1}}{s}\right)\left(i_{\mathrm{pr}^{\mathrm{ref}}}^{d} - i_{\mathrm{pr}}^{d}\right) \tag{6a}$$

$$u_{c^{\mathrm{ref}}}^{q} = u_s^q + \Delta u_c^q + \left(K_{p1} + \frac{K_{i1}}{s}\right)\left(i_{\mathrm{pr}^{\mathrm{ref}}}^{q} - i_{\mathrm{pr}}^{q}\right). \tag{6b}$$

After introducing two state variables, M_d and M_q, such that

$$\frac{dM_d}{dt} = K_{i1}\left(i_{\mathrm{pr}^{\mathrm{ref}}}^{d} - i_{\mathrm{pr}}^{d}\right) \tag{7a}$$

$$\frac{dM_q}{dt} = K_{i1}\left(i_{\mathrm{pr}^{\mathrm{ref}}}^{q} - i_{\mathrm{pr}}^{q}\right) \tag{7b}$$

the current controller equations become

$$u_{c^{\mathrm{ref}}}^{d} = u_s^d + \Delta u_c^d + K_{p1}\left(i_{\mathrm{pr}^{\mathrm{ref}}}^{d} - i_{\mathrm{pr}}^{d}\right) + M_d \tag{8a}$$

$$u_{c^{\mathrm{ref}}}^{q} = u_s^q + \Delta u_c^q + K_{p1}\left(i_{\mathrm{pr}^{\mathrm{ref}}}^{q} - i_{\mathrm{pr}}^{q}\right) + M_q. \tag{8b}$$

The actual value of the voltage lags the reference due to the time-lag of the converter’s power electronics. The relation between the actual value and the reference value can be represented by a time delay with time constant T_σ [14]:

$$\frac{U_c^{dq}}{U_{c^{\mathrm{ref}}}^{dq}} = \frac{1}{T_\sigma s + 1} \tag{9}$$

or in the time-domain

$$\frac{du_c^d}{dt} = \frac{1}{T_\sigma}\left(u_{c^{\mathrm{ref}}}^{d} - u_c^d\right) \tag{10a}$$

$$\frac{du_c^q}{dt} = \frac{1}{T_\sigma}\left(u_{c^{\mathrm{ref}}}^{q} - u_c^q\right). \tag{10b}$$

Combining the time-lag (10a) and (10b) and the equations of the current controllers (8a) and (8b) yields

$$\frac{du_c^d}{dt} = -\frac{K_{p1}}{T_\sigma}i_{\mathrm{pr}}^{d} - \omega\frac{L_{\mathrm{pr}}}{T_\sigma}i_{\mathrm{pr}}^{q} - \frac{1}{T_\sigma}u_c^d + \frac{1}{T_\sigma}M_d + \frac{K_{p1}}{T_\sigma}i_{\mathrm{pr}^{\mathrm{ref}}}^{d} + \frac{1}{T_\sigma}u_s^d \tag{11a}$$

$$\frac{du_c^q}{dt} = -\frac{K_{p1}}{T_\sigma}i_{\mathrm{pr}}^{q} + \omega\frac{L_{\mathrm{pr}}}{T_\sigma}i_{\mathrm{pr}}^{d} - \frac{1}{T_\sigma}u_c^q + \frac{1}{T_\sigma}M_q + \frac{K_{p1}}{T_\sigma}i_{\mathrm{pr}^{\mathrm{ref}}}^{q} + \frac{1}{T_\sigma}u_s^q. \tag{11b}$$

B. Outer Controllers

1) Reactive Power Control and Voltage Control: In a VSC HVDC system, every converter can independently control its reactive power injection in the power system. When the system voltage is aligned with the q-axis, the reactive power Q can be calculated as follows:

$$Q = u_s^q i_{\mathrm{pr}}^{d}. \tag{12}$$

The d-axis current setpoint, $i_{\mathrm{pr}^{\mathrm{ref}}}^{d}$, is thus calculated from the reactive power setpoint Q_{ref}. A combination of an open loop