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Modeling of inventory control with regenerative processes

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Abstract

In this paper we will discuss a general framework for single-item inventory models based on the theory of regenerative processes. After presenting without proof the main theorems for regenerative processes we analyze in detail how the different single-item models can be embedded within this general theory. This facilitates to write down the expressions for the average cost associated with an arbitrary costrate function *f*. Since these expressions are still complicated, involving convolutions, we use a recently developed numerically stable Laplace inversion algorithm to compute these objective functions in MATLAB©. This enables us to compute the costs instead of using approximations. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

Ford Harris' famous paper on the EOQ model in 1913 (cf. [1]) was the first of many publications on inventory theory. At present, thousands of papers have appeared in the management science and operations research literature. One may wonder why so much research is done on inventory models. The explanation is simply that, in practice, one encounters many different situations and each one requires a tailor-made analysis. For example, there may be differences with respect to the following aspects: number of locations and echelons, number of products, demand process, cost structure, service requirements and measurement, possible moments of placing a replenishment order, the way a stock-out is handled, and the lead time of replenishment orders. Since so many different situations can be analyzed, we feel that there is a need to develop a general framework. Such a framework will help to improve the understanding of the models that appeared in the literature. In this paper the average cost for a number of basic inventory models will be derived using a general framework which is presented in the next section. This framework is based on the theory of regenerative processes. It can be shown that most inventory models satisfy the so-called regenerative property which allows for a nice derivation of the average cost. Moreover, the newly developed algorithm for numerically inverting Laplace transforms (cf. [2]) enables us to calculate the exact costs, instead of using approximations. We will restrict ourselves to inventory systems with a single product, a single location, backordering of stockouts and deterministic lead times. In most of the inventory literature such a system is considered and for an overview the reader is referred to Chikán

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[3]. A more recent discussion of those models is given by de Kok [4], usually the analysis of those models serve as a basis for multi-location, multi-echelon and multi-item systems. The theory of regenerative processes is briefly discussed in Section 2 and its application to inventory models is presented in Section 3. Section 4 deals with a more detailed discussion of the different classical single-item inventory models. In this section the exact expression of the average cost for each model is presented. In Section 5 the algorithm for numerically inverting Laplace transforms is applied to a numerical example of the so-called (R, S) inventory model. This is the simplest of the considered models and in a forthcoming paper we will show how to apply this algorithm to the other more complicated models. Finally, in Section 6 some conclusions are presented.

2. Regenerative processes

The theory of regenerative processes plays a major role within stochastic models. For these stochastic processes many properties and results are presented in the book of Asmussen [5]. Since for our purpose it is sufficient to consider only pure regenerative processes, we will not discuss delayed regenerative processes. The goal of this section is to give an overview of the theory of pure regenerative processes and we first start with a simplified version of a pure regenerative process. Observe that random variables are denoted by boldface characters while the set T either denotes the set $[0, \infty)$ or $\mathbb{N} \cup \{0\}$.

Definition 2.1. A stochastic process $\boldsymbol{X} = \{\boldsymbol{X}(t)\colon t \in T\}$ with metric state space E is called a pure regenerative process if there exists some positive constant $\sigma \in T$ such that for every $n \in \mathbb{N} \cup \{0\}$ the distribution of the shifted stochastic process $\{\boldsymbol{X}(t + n\sigma)\colon t \in T\}$ is independent of n.

A more general definition of a pure regenerative process is given by the next one.

Definition 2.2. A stochastic process $\boldsymbol{X} = \{\boldsymbol{X}(t)\colon t \in T\}$ with metric state space E is called a pure regenerative process if there exists an increasing sequence $\boldsymbol{\sigma}_n$, $n \in \mathbb{N} \cup \{0\}$ with $\boldsymbol{\sigma}_0 := 0$ of random points belonging to T, satisfying

1. The random variables $\boldsymbol{\sigma}_{n+1} - \boldsymbol{\sigma}_n$, $n \in \mathbb{N} \cup \{0\}$, are independent and identically distributed with right continuous cumulative distribution function F_σ satisfying $F_\sigma(0) = 0$ and $F_\sigma(\infty) = 1$.
2. For each $n \in \mathbb{N} \cup \{0\}$ the post-$\boldsymbol{\sigma}_n$ process

   $\{\boldsymbol{X}(t + \boldsymbol{\sigma}_n)\colon t \in T\}$

   is independent of $\boldsymbol{\sigma}_0, \ldots, \boldsymbol{\sigma}_n$.
3. The distribution of $\{\boldsymbol{X}(t + \boldsymbol{\sigma}_n)\colon t \in T\}$ is independent of n.

In case the difference $\boldsymbol{\sigma}_{n+1} - \boldsymbol{\sigma}_n, n \in \mathbb{N} \cup \{0\}$, is degenerate and given by $\sigma = \boldsymbol{\sigma}_{n+1} - \boldsymbol{\sigma}_n > 0$ with probability one then it is obvious that the definition of a pure regenerative process as mentioned in Definition 2.1 reduces to Definition 2.1. Moreover, in most applications with $T = [0, \infty)$ a pure regenerative process $\boldsymbol{X}$ is càdlàg. This means that the sample paths of the stochastic process $\boldsymbol{X}$ are right continuous with left limits $\mathbb{P}$-almost surely with $\mathbb{P}$ denoting the probability measure of the underlying probability space. For pure regenerative processes the next result is easy to verify.

Theorem 2.1. *If the stochastic process $\boldsymbol{X} = \{\boldsymbol{X}(t)\colon t \in T\}$ is a pure regenerative process with metric state space E and increasing sequence $\boldsymbol{\sigma}_0 < \boldsymbol{\sigma}_1 < \cdots$ and $\Phi\colon E \to \mathbb{R}$ is a Borel measurable function, then the process $\Phi \circ \boldsymbol{X} := \{\Phi(\boldsymbol{X}(t))\colon t \in T\}$ is a pure regenerative process with the same increasing sequence. Moreover, if $T = [0, \infty)$ and Φ is a continuous function then the stochastic process $\Phi \circ \boldsymbol{X}$ is càdlàg if $\boldsymbol{X}$ is càdlàg.*

To introduce a cost structure on a pure regenerative process we consider a nonnegative Borel measurable function $f\colon E \to \mathbb{R}$, called the costrate function. Using this function we denote by $\boldsymbol{C} = \{\boldsymbol{C}(t)\colon t \in T\}$, with $T = [0, \infty)$, the stochastic cumulative cost process given by $\boldsymbol{C}(t) := \int_0^t f(\boldsymbol{X}(s))\,\mathrm{d}s$, while for $T = \mathbb{N} \cup \{0\}$ it is given by $\boldsymbol{C}(t) := \sum_{n=0}^{t} f(\boldsymbol{X}(n))$. We always assume that this stochastic cumulative cost process $\boldsymbol{C}$ is well defined and that $\mathbb{E}\boldsymbol{C}(t) < \infty$ for every $t \in T$. To limit the size of this paper, we only consider pure