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Optimal Policies in Continuous Time Inventory Control Models with Limited Supply

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Abstract—In this paper, we deal with the problem of a fixed number of units of a certain perishable commodity over a continuous time horizon. Airline seats, hotel rooms, advertising space in newspapers, and some seasonal products that must be sold before the end of the season are examples of such commodities that cannot be carried over and are not storable for consumers. This paper considers such a problem of continuous time perishable inventory control by applying semi-Markov decision processes over a finite time horizon. It is shown that there is an optimal policy that is simple and stationary. Furthermore, some analytical properties of this optimal policy and its value function are explored under certain specific assumptions.

Keywords—Inventory control, Perishable products, Yield management, Semi-Markov decision, Contraction mapping.

1. INTRODUCTION

Consider a certain perishable commodity whose supply is fixed. A company wants to sell a fixed number of units of this commodity. On the other hand, consumers have different (nonhomogeneous) preferences for the commodity. In such a circumstance, the company can set different prices on the identical commodity; that is, the law of one price does not hold in this market. Such commodities are called price differentiation products. Examples of price differentiation products might include airline seats, hotel rooms, concert tickets, cabins on cruise ships, and seasonal goods such as winter coats that have to be sold before the end of the season. The date and place for selling such commodities is fixed, it is not possible to use these commodities either before or after the specified date, and they are perishable goods that cannot be carried over. It is possible, however, to sell these commodities beforehand.

In this paper, we develop a framework of inventory control for perishable commodities with limited supply by explicitly incorporating the existence of multiple prices for that commodity. It is shown that such an inventory control model can be formulated as a semi-Markov decision process over a finite continuous time horizon. Our main result is to show that, under some specific assumptions, there is an optimal policy and an optimal value function corresponding to it. In addition, some analytical properties are explored.

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The problem of selling a fixed number of units of a certain perishable commodity in order to maximize the expected revenue is known as the yield management problem in the airline industry (see [1–3]), where most studies on yield management have focused on either pricing policy or static control problems with two fares (see [1,4–7]). Feng and Gallego [8] consider the problem of deciding the optimal timing of a single-price change from a given initial price to either a lower or higher price. Gallego and Ryzin [5] investigate the problem of dynamic pricing policies for selling a given stock of items by a deadline. Lee and Hersh [9] develop a discrete-time dynamic programming model for finding an optimal booking policy for airline seat inventory control. Brumelle and Walczak [10] model an airline seat allocation for a single-leg flight with multiple fares that is similar to ours but has different assumptions. The present study differs from these earlier studies in two regards. First, the planning horizon is continuous times; second, ours is a dynamic model that deals with a multiple-price selling policy for a perishable product that is not necessarily restricted to airline seats.

In Section 2, we develop the problem of selling a fixed number of units of a perishable commodity over a continuous time planning horizon. We apply a semi-Markov decision process approach to the problem so as to maximize the expected total revenue. Section 3 discusses what analytical properties an optimal policy and its value function possess. In particular, we are interested in the development of value function concavity and optimal policy monotonicity with respect to their arguments; these tell us that an optimal selling curve decreases as the time draws nearer to the end of the planning horizon. Finally, the last section is a conclusion with additional comments.

2. PERISHABLE INVENTORY CONTROL WITH FIXED SUPPLY

Suppose that a firm wants to sell a fixed number of units of a certain perishable commodity that cannot be carried over and is not storable for consumers. Let C be the total number, fixed and given, of such a saleable inventory. The commodity can be sold in advance at different multiple prices. Whenever a customer arrives, he/she declares his/her price and the demand size desired. Hence, customers can be described by their price and demand size requests. Previous models are either single-period models in discrete time, or the demand is of two kinds and independent, and they assumed that demand for a low price would materialize faster than the demand for a high price (the early bird assumption). Our model does not require such an assumption.

Let time 0 be the point at which sales begin and time T the point at which sales end; also, let the planning horizon be the closed interval of continuous time $[0,T]$. The price is of the K type; let the k^{th} price be π_k, and $\pi_1 > \pi_2 > \cdots > \pi_K$. Let s be the size of the demand at the time the customer arrives. We assume that $0 \le s \le S$. Let us put the pair $(k,s) \equiv \phi$, and call ϕ the demand type. This means that all the demand types are of the $K \cdot S$ kind, and let this set be $M = \{\phi \mid \phi = (k,s) \in K \otimes S\}$. If we let the time t_n be the time of arrival of the n^{th} demand and the random variable Φ_n be the demand type at the time t_n, then the stochastic process $\{\Phi_n, t_n : n = 1,2,\dots\}$ will be a semi-Markov process. In other words,

$$\begin{aligned} &P_r\left\{\Phi_{n+1} = \phi',\ t_{n+1} - t_n \le v \mid \Phi_0,\dots,\Phi_n; t_0, t_1,\dots,t_n\right\} \\ &\quad = P_r\left\{\Phi_{n+1} = \phi',\ t_{n+1} - t_n \le v \mid \Phi_n, = \phi,\ t_n = u\right\} \\ &\quad \equiv P\left(\phi', v \mid \phi, u\right). \end{aligned} \tag{2.1}$$

Equation (2.1), which is a conditional joint probability of state transition and the time that elapses before the next transition time, usually depends on the commodity's sales policies. In the present study, however, we assume that the demand arrival process is independent of the commodity's sales policies. Next, in order to remove the possibility of the arrival of unlimited numbers of demands within any subinterval of the planning horizon, we set up the following assumption.