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A simple decision rule for decentralized two-echelon inventory control

Sven Axsäter*

Department of Industrial Management and Logistics, Lund University, P.O. Box 118, Lund 22100, Sweden

Abstract

We consider a two-echelon distribution inventory system with a central warehouse and a number of retailers. The retailers face stochastic demand. The system is controlled by continuous review installation stock (R, Q) policies with given batch quantities. One way to decentralize the system control is to provide a backorder cost to the warehouse, and let the warehouse choose its reorder point so that the sum of the expected holding and backorder costs are minimized. Given the resulting warehouse policy, the retailers similarly optimize their costs with respect to the reorder points. This paper provides a simple approximate technique for determining the backorder cost to be used by the warehouse.

Keywords: Inventory management; Two-level; Stochastic demand; Decentralized supply chain

1. Introduction

A two-echelon distribution inventory system consisting of a central warehouse and a number of retailers is considered (see Fig. 1). The retailers face independent stochastic demand processes. (In our numerical study we consider pure Poisson demand as well as compound Poisson demand.) All sites apply continuous review installation stock (R, Q) policies (or (R, nQ) policies). An (R, Q) policy means that when the inventory position at a considered installation declines to or below the reorder point R, a number of batches of size Q are ordered such that the resulting inventory position after ordering is in the interval $(R, R+Q]$. The inventory position is the stock on hand, plus outstanding orders, and minus backorders. The batch quantities are assumed to be given. They may, for example, have been determined in a deterministic model. Our goal is to determine suitable reorder points under a standard cost structure with linear holding costs at all sites and linear backorder costs or, alternatively, fill rate constraints at the retailers. (Fill rate = fraction of demand that can be satisfied immediately from stock on hand.)

The considered two-echelon problem can be solved exactly or approximately under various conditions. Overviews of different techniques are

*Tel.: +46-46-222-3387; fax: +46-46-222-4619.
E-mail address: sven.axsater@iml.lth.se (S. Axsäter).

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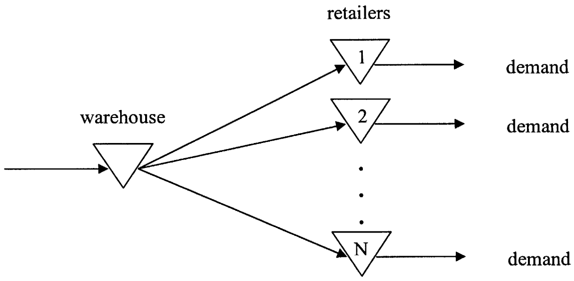

Fig. 1. Two-echelon distribution inventory system.

given in Axsäter (1993), Federgruen (1993), and in the recent textbooks Axsäter (2000b), and Zipkin (2000). However, such techniques are usually difficult to implement in practice because of their complexity. In practice, it is normally more attractive to decentralize the control and let each installation apply single-level techniques. This means that we need to introduce a cost structure for the warehouse that appropriately reflects the cost consequences of warehouse delays for the retailers. One possibility is to supply a standard backorder cost to the warehouse, and let the warehouse optimize the sum of its expected holding and backorder costs with respect to its reorder point. Given the resulting warehouse policy, the retailers can, similarly, optimize their costs with respect to their reorder points. This paper provides a simple approximate technique for determining the backorder cost to be used by the warehouse. The suggested procedure is related to the more complex methods in Andersson et al. (1998) and Axsäter (2001), and also to the METRIC technique by Sherbrooke (1968). A general idea is that the procedure should be based on standard single-echelon techniques and be easy for practitioners to understand.

A suitable backorder cost for the warehouse does, of course, not solve the multi-echelon inventory problem completely. Given the backorder cost, the warehouse still has to optimize its total costs under a complex demand process, i.e., the orders from the retailers. Furthermore, given the warehouse policy, the retailers must minimize their costs under the stochastic lead-time variations caused by the shortages at the warehouse. In this paper, we disregard these remaining problems and focus only on the determination of the warehouse backorder cost.

The outline of the paper is as follows. In Section 2 we give a detailed problem formulation. Section 3 describes our procedure for determination of the warehouse backorder cost. A numerical evaluation of the technique is presented in Section 4. Finally, we give some concluding remarks in Section 5.

2. Problem formulation

It is assumed that the retailers face stationary stochastic demands, which are independent across retailers and over different times. For each retailer we know the mean and standard deviation of the demand per unit of time. All installations apply continuous review (R, Q) policies. The batch quantities are given but we need to determine the reorder points. Demand that cannot be met directly is backordered at all locations. We wish to minimize the expected total variable costs, i.e., the sum of the holding costs at all locations and the backorder costs at the retailers. Let us introduce the following notation:

$N =$ number of retailers,
$L_0 =$ constant lead-time for an order to arrive at the warehouse from the outside supplier,
$L_i =$ constant transportation time for an order to arrive at retailer i from the warehouse,
$\mu_i =$ average demand per unit of time at retailer i,
$\mu_0 = \sum_{i=1}^{N} \mu_i =$ average demand per unit of time at the warehouse,
$\sigma_i =$ standard deviation of demand per unit of time at retailer i,
$Q_0 =$ batch size at the warehouse,
$Q_i =$ batch size at retailer i,
$R_0 =$ warehouse reorder point, $R_0 \geqslant -Q_0$,
$R_i =$ reorder point for retailer i, $R_i \geqslant -Q_i$,
$h_i =$ holding cost per unit and time unit at retailer i,