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Analytical function describing the behaviour of a thermocline storage tank: A requirement for annual simulations of solar thermal power plants

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Thermocline tank behaviour during dynamic processes of charge/discharge and stand-by periods has been described in dimensionless form by means of the Logistic Cumulative Distribution Function (CDF). The Logistic-CDF parameter, *S*, has been correlated with both thermocline tank features and operation conditions (i.e. dimensionless velocity, υ^*, and time, t^*) with the help of a single-phase one-dimensional numerical model previously developed and validated. For the dynamic processes, it has been found that *S* becomes independent on υ^* when $\upsilon^* > 2500$, but it always increases proportionally with $\sqrt{\upsilon^* t^*}$ due to the progress of thermocline zone inside the tank. For adiabatic stand-by periods, in which $\upsilon^* = 0$, *S* parameter proportionally increases with $\sqrt{t^*}$ whereas z_c^* varies in order to keep constant the stored energy inside the tank. In terms of thermocline thickness it has been demonstrated that it is proportional to *S* parameter but its exact value strongly depends on the temperature accuracy that determines thermocline zone limits. This parameterised Logistic-CDF provides a general analytical equation for the outlet temperature of the storage liquid, which allows implementing thermocline storage systems in the annual performance simulations of solar thermal power plants.

1. Introduction

One of the main advantages of solar thermal power plants (STPP's) with thermal energy storage (TES) systems is their feasibility for energy dispatching whenever it is required. Up to the present, the molten salt two-tank system [1,2] has been the storage option most widely implemented in commercial STPP's. However, since the estimated cost of this kind of storage is still very high (about 30–50 US$/kW$_{th}$h [3]), strong efforts are currently being made for developing other storage concepts with high cost reduction potential [4]. One of these concepts is the thermocline single-tank storage [5–8], which, according to the Electric Power Research Institute (EPRI), could reduce around 33% the cost of the TES system [9,10]. However, despite their great potential for decreasing thermal storage costs, thermocline tanks still present many challenges in terms of control, power plant integration and efficiency.

From a practical point of view one way to have a good picture of thermocline tank behaviour, is to simulate the annual performance of a STPP in which this kind of storage has been implemented. Several simulation models for parabolic trough plants that include thermal storage systems have been already developed by different organisations [11–15]. All these models only consider the conventional molten-salt two-tank storage system for which outlet/inlet temperatures are fixed by design and the power supplied/extracted is easily calculated through the molten salt mass flow rate. However, in a thermocline storage tank, the available thermal energy at maximum temperature is expected to decrease with subsequent charging/discharging cycles [16] due to a continuous increase of thermocline thickness. Moreover, thermocline thickness also increases during idle periods because of thermal diffusion [17], which decreases TES exergy and hence the potential useful power that can be extracted.

Fig. 1 illustrates the difference between both two-tank and thermocline storage systems in terms of outlet temperature as a function of time for the particular case of an initial discharge process. While the two-tank system provides maximum constant storage liquid temperature, T_{max}, during the whole discharging process until t_{end}; liquid outlet temperature for a thermocline tank starts decreasing after a certain time, $t_{partial\text{-}discharge}$ down to the minimum temperature, T_{min} when full discharge time, $t_{full\text{-}discharge}$ is attained. This special behaviour of thermocline tanks makes it difficult to implement them as storage systems in STPP's simulations.

Up to now, thermocline storage tanks meant to be used in STPP's have been widely simulated by means of different numerical models, but due to their complexity and long computing time,

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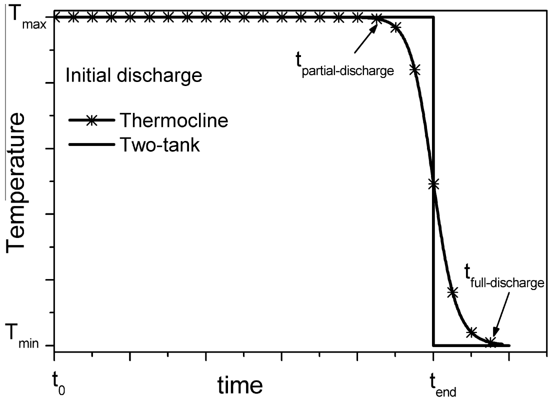

Fig. 1. Outlet temperature profile as a function of time for initial discharge processes corresponding to both a two-tank storage system and a single thermocline storage tank.

those models account only for the tank behaviour [8,26,18–23]. Actually, the only attempt to integrate a thermocline TES in a STTP model has been made by Kolb et al. [24,25], who included a thermocline-tank numerical model as a TRNSYS component in the whole plant configuration. In their calculations, the computing time for the annual plant performance was not significantly increased because they considered very few control volumes (i.e. 23) in the tank simulation. However, as demonstrated in our previous work [26], the use of only 23 control volumes in a thermocline tank numerical simulation leads to results with low accuracy and so are expected to be the annual performance calculations of the corresponding power plant.

In this way, the aim of this work is to describe the thermocline tank behaviour by means of an analytical function which accurately provides outlet temperature with time and can be implemented in any kind of code used for simulating the annual performance of a STPP. Assuming single-phase one-dimensional models, various authors have obtained analytical solutions for the corresponding differential energy balance equation describing a thermocline tank. Some of them have solved the equation by using Laplace transforms [27–29] or by means the Greens' function methodology [30]; while others have proposed polynomial approximate expressions [9,31]. All these analytical solutions consist of sigmoid curves representing temperature distribution along the tank height for different time values. In the particular case of the solutions obtained by Laplace transform, the curves are represented by the Normal Cumulative Distribution Function (Normal-CDF) [32]. In this work the Normal-CFD has been substituted by the Logistic-CDF because this function has already been proposed by some authors as a good approximation to the Normal-CDF [33]. In order to obtain general results valid for any kind of thermocline tank, the Logistic-CDF has been expressed in dimensionless coordinates. Furthermore the Logistic-CDF parameter, *S*, has been correlated with tank features and working conditions by fitting the function to the numerical results calculated with the single-phase one-dimensional model developed in our previous work [26]. With the help of these correlations, the behaviour of thermocline tank has been analytically described not only during dynamic charge/discharge processes but also during stand-by periods.

2. Results and discussion

2.1. Analytical function describing thermocline tank behaviour

The majority of analytical solutions found in the literature for thermocline tanks have been obtained for single-phase one-dimensional models expressed in dimensionless form. In some cases the equation describing temperature distribution along the tank height has been represented by the complementary error function [27,28], which can be directly related to the Normal Cumulative Distribution Function (Normal-CDF) [32]. However, since this function implies solving an integral, its calculation is usually avoided and various authors have used more simple polynomial approximations [9,31]. In this paper the Logistic-CDF [34] is proposed as the alternative approximation to Normal-CDF for describing the thermal behaviour of thermocline storage tanks. Actually, the replacement of Normal-CDF by the Logistic-CDF has been already suggested by some authors in order to simplify calculations in the field of statistics [33]. The general equations for both CDF's are:

$$\text{Normal-CDF}: \; y(x;\mu,\sigma) = y_{\min} + \frac{y_{\max}-y_{\min}}{2}\left[1+erf\left(\frac{x-\mu}{\sqrt{2\sigma^2}}\right)\right] \quad (1)$$

$$\text{Logistic-CDF}: y(x;\mu,S) = y_{\min} + \frac{y_{\max}-y_{\min}}{1+e^{-(x-\mu)/S}} \quad (2)$$

where $y_{\max}$ and $y_{\min}$ are respectively the uppermost and the lowest limits of the dependent variable *y*, and μ represents the mean but also the *x* value where both CDF curves have their inflection point. The parameter σ^2 in the Normal-CDF corresponds to the variance whereas in the Logistic-CDF, the *S* parameter is related to the variance through the expression:

$$\text{Variance} = \sigma^2 = \frac{\pi^2}{3}S^2 \quad (3)$$

In Fig. 2 Normal and Logistic CDF functions are represented for the case in which $y_{\min} = 0$, $y_{\max} = 1$, $\mu = 0.5$, $\sigma = 0.01$ and hence $S = 0.055$. In this Figure we can clearly see that the two curves are very close, being the maximum difference between them of about ±0.02.

2.2. Logistic-CDF parameterisation for thermocline dynamic processes

In previous works [26,23] we presented a single-phase one-dimensional numerical model (called CIEMAT1D1SF) for simulating thermocline storage tanks with an effective storage medium formed by either a liquid or both a liquid and a solid filler. This model neglected thermal losses, was expressed in dimensionless coordinates and its numerical results were successfully validated with experimental data found in the literature for different kinds of thermocline storage tanks [8,35,36].

CIEMAT1D1SF model was developed for solving the following differential energy balance equation describing the dynamic processes of charge and discharge for a thermocline storage tank:

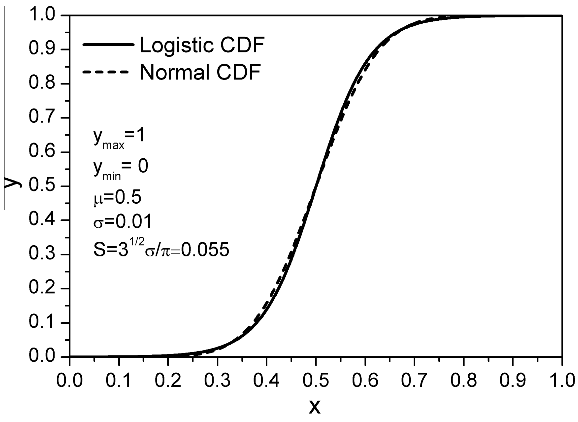

Fig. 2. Comparison of Normal-CDF and Logistic CDF curves for the case in which $y_{\min} = 0$, $y_{\max} = 1$, $\mu = 0.5$, $\sigma = 0.01$ and $S = 0.055$.