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Production and inventory control with chaotic demands

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Abstract

This study explores an efficient approach for identifying chaotic phenomena in demands and develops a production lot-sizing method for chaotic demands. Owing to the butterfly effect of chaotic demands, precise prediction of long-term demands is difficult. The experiments conducted in this study reveal that the maximal Lyapunov exponent is very effective in classifying chaotic and non-chaotic demands. A computational procedure of the Lyapunov exponent for production systems has been developed and some real world chaotic demands have been identified using the proposed chaos-probing index. This study proposes a modified Wagner–Whitin method that uses a forward focused perspective to make production lot-sizing decision under chaos demands for a single echelon system. The proposed method has been empirically demonstrated to achieve lower total production costs than three commonly used lot-sizing models, namely: lot-for-lot method, periodic ordering quantity, and Silver-Meal discrete lot-size heuristic under a fixed production horizon, and the conventional Wagner–Whitin algorithm under chaotic demands. Sensitivity analysis is conducted to compare changes in total cost with variations in look-ahead period, initial demand, setup cost and holding costs.

Keywords: Production and inventory control; Chaotic demands; Lot sizing

1. Introduction

Facing a changing market environment, the most difficult parameter to manage is adjusting production to meet market demand. This study derives an economic production plan under chaotic demands to minimize overall cost.

Recent studies have showed that when demand is lumpy, order quantity changes significantly between periods. The traditional economic order quantity (EOQ) models cannot be applied to solve these types of production problems [1]. Moreover, other sophisticated algorithms such as the Wagner–Whitin dynamic technique, have not been successful in solving these types of problems due to the extreme sensitivity of the solution to changes in the estimates of future demands. Carlson et al. [2] presented a solution procedure, which incorporates the cost of changing the current production schedule to alleviate such nervousness in the face of fluctuating demand. Previous studies such as Backburn and Millen [3] and Zhao et al. [4] describe lot-sizing approaches for rolling and fixed time horizons. However, lot-sizing approaches for dealing with chaotic demand remain in their infancy.

This study considers the butterfly effect of chaotic demands, and develops a corresponding production lot-sizing strategy. Researchers have long lacked methods for describing or analyzing chaotic natural environments, for example, weather changes and the wave motions in an ocean. A chaotic model has a special characteristic in that the starting value strongly influences system behavior. Small drift in predicting an initial demand ultimately may cause a significant difference to real demand. This phenomenon is normally called the "butterfly effect". An example of this effect

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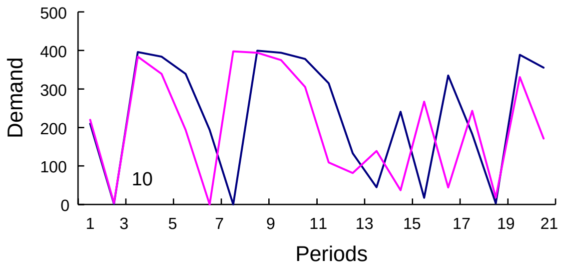

Fig. 1. Butterfly effect in chaotic demands.

is shown in Fig. 1. For production systems facing chaotic demands, the butterfly effect implies that a production schedule based on a long-term ‘accurate’ prediction may be useless given chaotic demand. Therefore, this study recommends using a production plan with a short-term planning horizon.

The butterfly effect in chaotic demands makes precise prediction of long-term demands difficult. Moreover, even when long-term demand is predictable, they can be distorted dramatically owing to the butterfly effect. In such a case, long-term production planning may damage the system because of incorrect demand information. Therefore, the production horizon should be divided into segments to reduce computational effort and increase lot-sizing decision accuracy. This study identifies the chaotic demands and formulates a production lot-sizing strategy for the system.

This paper is organized as follows: Section 2 presents an efficient approach for identifying chaotic demands. Section 3 then introduces several approaches for the deterministic dynamic lot-sizing model with lumpy demand models for lumpy demands. Subsequently, Section 4 proposes a heuristic production lot-sizing strategy for dealing with chaos demands. Next, Section 5 compares the total production costs of the proposed method and some other widely used approaches for different chaotic demand types. Finally, Section 6 examines the validity of the method developed here, and demonstrates how changes in planning horizon, initial value, setup cost and unit holding costs influence total production costs.

2. Identifying chaotic demands

Chaotic phenomenon can be either qualitatively identified by figure patterns such as Poincare map, or by quantitative measures, such as various dimension merits [5], Kolmogorov entropy [6], and Lyapunov exponent [7]. Among these figure patterns, Lyapunov exponent is computationally simple and easy to use. Provided the maximal Lyapunov exponent is a positive number, the investigated demands are likely to be chaotic. This study employs the maximal Lyapunov exponent to identify chaotic demands.

However, the original design of the Lyapunov exponent is used for nonlinear and continuous dynamic systems, while in the production systems being investigated, demands are usually for discrete quantities. The Lyapunov exponent is defined in Section 2.1 and an adaptation of the computational procedure for the Lyapunov exponent is presented. Section 2.2 provides some identified practical chaotic demands.

2.1. Preliminaries of Lyapunov exponent

Assuming a demand series, $x(t)$, which is constructed within an m-dimension space, an individual demand is located in the space at the coordination of $\{x(t), x(t+\tau), \ldots, x(t+[m-1]\tau)\}$, where τ is the time lag (for detail description, refer to [8]). The analysis presented here first identifies the starting point A_0 (in the space) which is closest to the initial reference point, t_0. The distance between the two points is defined as $L(t_0)$ (Fig. 2). After an evolution time period, the fiducial trajectory (which is consisted of all reference points) reaches t_1, and the neighbor trajectory reaches A_1'. At this moment, the distance between the two points becomes $L'(t_1)$. This study then identifies a new start point according to the following two principles:

- The distance between point A_1 and the new reference point t_1 should be below a predefined small value.
- The angle formed by points A_1, t_1 and A_1' should be less a predefined value.

If an initial point A_1 cannot be found using the above two principles, the system should proceed to the next evolved reference point, t_2, and find an appropriate initial point, A_2, which satisfies the above principles. This procedure continues until all data points constructed by the demand series are reviewed. The maximal Lyapunov exponent $\lambda_{\max}$ is defined as

$$\lambda_{\max} = \frac{1}{t_M - t_0} \sum_{k=1}^{M} \log_2 \frac{L'(t_k)}{L(t_{k-1})}, \tag{1}$$

where M denotes the times of finding a new initial point.

The demand series must be transformed into a scope between zero and 1 as the maximal Lyapunov exponent is applied. Eq. (2) is employed to do the normalization.

$$\frac{x_n - \text{minimal demand}}{\text{maximal demand} - \text{minimal demand}}. \tag{2}$$

When calculating the maximal Lyapunov exponent, the parameters of embedded dimension, reconstruction delay time, data sampling time interval, and evolution time must be calculated using other methods. This study sets the embedded dimension, reconstruction delay time, data sampling time interval, and evolution time as those employed in Rosenstein et al. [7].

A demand series originally proceeds along a single dimension. One requires confirming the most appropriate dimension size so that the estimation of the maximal Lyapunov exponent can be accurate. Fig. 3 proposes a procedure for obtaining a better estimation of the dimension